SOLUTIONS - CLASS X

CHAPTER-1

REAL NUMBERS

1 $\frac{147}{120} = \frac{49}{40} = \frac{49\times5^2}{2^3\times5^1\times5^2} = \frac{49\times25}{10^3} = \frac{1225}{1000}$ =1.225

Hence the decimal expansion of $\frac{147}{120}$ will terminate after three places of decimal.

HCF$\times$LCM=Product of numbers.

$15\times$LCM=105×120

LCM= $\frac{105\times120}{15}$ = 840 *Ans*

2 21975= $3\times5\times5\times293$
= $3\times5^2\times293$

3 $2^3\times3^2\times5^2\times7$
=$2^2\times5^2\times2^1\times3^2\times7$

= $(10)^2\times2\times3^2\times7$

since 10^2=100

Hence no. of zeroes = 2

5 $\frac{64}{455}$

455= $5\times7\times13$

Since prime factors of denominator are other than $2^n\times5^m$, therefore $\frac{64}{455}$ will have non-terminating-repeating decimal expansion.

LEVEL-2

1 Let $\frac{2\sqrt{3}}{5}$ be a rational number.

Then $\frac{2\sqrt{3}}{5} = \frac{p}{q}$, where p and q are co-primes and q$\neq$ 0

Now $\sqrt{3} = \frac{p}{q}\times\frac{5}{2}$

Here RHS is rational where LHS is irrational

$\therefore$. Our supposition is wrong.

Hence $\frac{2\sqrt{3}}{5}$ is an irrational number.

2 Let 5-$\sqrt{2}$,be a rational number.

Then, 5-$\sqrt{2}$ = $\frac{p}{q}$, where p and q are co-primes and q $\neq$ 0

Then 5- $\frac{p}{q}$ = $\sqrt{2}$

$\Rightarrow \frac{5q-p}{q}$ = $\sqrt{2}$

Here LHS is rational but RHS is Irrational.

$\therefore$ Our assumption is wrong.

Hence 5-$\sqrt{2}$ is an irrational number.

3 7$\times$11$\times$13$\times$17+17

=17$\left(7\times11\times13+1\right)$

=17$\left(1001+1\right)$

= 17$\times$1002

= 17034

Therefore, by Fundamental Theorem of Arithmetic, 17$\times$1002 is a composite number.

4 Let the three consecutive positive integers be n, n+1, n+2.

Now n is of the form 3q, 3q+1, 3q+2.

Case 1 n=3q

Here n is divisible by 3, but n+1, n+2 are not divisible by 3.

Case 2 n=3q+1

n+2= 3q+1+2= 3$\left(q+1\right)$ is divisible by 3 , but n and $\left(n+1\right)$ are not divisible by 3.

Case 3 when n= 3q+2

Here n+1= 3q+2+1= 3(q+1) is divisible by 3, but n and (n+2) are not divisible by 3.

Hence, one of n, n+1 and n+2 are divisible by 3.

LEVEL 3

1 We know that any positive integer is of the form 2m or 2m+1 for some positive integer m.

When n=2m, then

$n^2 - n = (2m)^2 - 2m$

$= 4m^2 - 2m$

$= 2m(2m-1)$

$=2q$, where $q = m(2m-1)$

$\Rightarrow n^2 - n$ is divisible by 2.

When n= 2m+1, then

$n^2 - n = (2m+1)^2 - (2m+1)$

$=(4m^2 + 4m+1) -2m \_1$

$=4m^2 + 2m$

$=2m(2m+1)$

$=2q$, where $q= m(2m+1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Q2 $45=27\times1+18$ eq(1)

$27=18\times1+9$ eq(2)

$18=9\times2+0$ eq(3)

from eq(2)

$9=27-18\times1$

$9=27-(45-27\times1)\times 1$

$=27-45\times1+27\times1\times1$

$=27-45+27$

$=54-45$

$9=27\times2-45\times1$

Compairing with d= 27x+45y

We get d=9, x=2, y= -1.

3 $HCF_{(117,72)} = 9$

Total number of books = 117

$\therefore$ Least number of bundles= $\frac{117}{9}$ = 13 bundles.

4 $117= 65\times1+52$

$65= 52\times1+13$

$52= 13\times4+0$

$13 = 65 - 52 \times 1$

$= 65 - (117 - 65 \times 1) \times 1$

$= 65 - 117 \times 1 + 65 \times 1 \times 1$

$13 = 65 \times 2 - 117 \times 1$

$13 = 65 \times x - 117 \times y$

where $x=2$, $y = -1$

5 $445 - 4 = 441$

$572 - 5 = 567$

$699 - 6 = 693$

By Euclid`s Division Algorithm

$567 = 441 \times 1 + 126$

$441 = 126 \times 3 + 63$

$126 = 63 \times 2 + 0$

$693 = 63 \times 11 + 0$

Hence , 63 is the largest number.

6 Any odd positive integer of the form 4q+1 or 4q+3 for some integer q

$\therefore$, we have following cases

CASE 1 When $n= 4q+1$

$n^2 - 1 = (4q+1)^2 - 1 = 16q^2 + 8q + 1 - 1$

$= 16q^2 + 8q = 8q(2q+1)$

$\Rightarrow$ $n^2 - 1$ is divisible by 8

CASE II When $n = 4q + 3$

$n^2 - 1 = (4q+3)^2 - 1 = 16q^2 + 24q + 9 - 1$

$= 16q^2 + 24q + 8$

$n^2 - 1 = 8(2q^2 + 3q + 1) = 8(2q+1)(q+1)$

$n^2 - 1$ is divisible by 8.

Hence $n^2 - 1$ is divisible by 8, if n is an odd positive integer.

POLYNOMIALS

LEVEL-1

Sol.1 $f(x) = x^3 - 2x^2 + 4x + k$

$\because$, x=1 is a zero of a polynomial

$\therefore$, f(1) = 0

$(1)^3-2(1)^2+4(1)+k = 0$

Or $1- 2+4+k=0$

Or $3+k = 0$

Or $k =-3$

Sol.2 Let $p(x)= x^2-x-(2k+2)$

∵ -4 is a zero of polynomial

∴ $p(-4)=0$

$(-4)^2-(-4)-(2k+2)=0$

or $16+4 -2k -2=0$

or $-2k+18=0$

or $K=9$

Sol.3 Let $p(x)= (x-2)(x-3)$

$p(3) = (3-2)(3-3)=0$

Yes ,3 is zero of given polynomial

$P(2)= (2-2)(2-3)= 0$

Yes, 2 is the zero of given polynomial.

Sol.4 $f(x)= 4x^2+8x$

$= 4x(x+2)$

So, the value of $4x^2+8x$ is zero when either $x=0$ or $x=-2$

∴ , the zeros of $4x^2+8x$ are 0 and -2.

Sum of zeros $=(-2)+0 = \frac{-2}{1} \times \frac{4}{4} = \frac{-8}{4} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

Product of zeros $= (0) \times (-2) = 0 = \frac{0}{1} = \frac{(constant\ term)}{coefficient\ of\ x^2}$

Sol. 5 (i) Let the quadratic polynomial be ax^2+bx+c

Let α,β be its zeros

$\alpha + \beta = \frac{1}{4}$, $\alpha\times\beta = -1$

or $\frac{-b}{a} = \frac{1}{4}$, $\frac{c}{a} = \frac{-1}{1} \times \frac{4}{4}$

or $\frac{b}{a} = \frac{-1}{4}, \frac{c}{a} = \frac{-4}{4}$

$\therefore,\ a = 4 , b = -1, c = -4$

$\therefore, required\ polynomial = 4x^2-x-4$

(ii) Let the quadratic polynomial be ax^2+bx+c

$\alpha +\beta = \sqrt{2}$, $\alpha\beta = \frac{1}{3}$

$\frac{-b}{a} = \frac{\sqrt{2}}{1} \times \frac{3}{3} , \frac{C}{a} = \frac{1}{3}$

$\therefore, a = 3, b = -3\sqrt{2} , c = 1$

$\therefore, required\ polynomial = 3x^2-3\sqrt{2}\ x+1$

LEVEL-2

Sol.1 on dividing $6x^3+13x^2+x-2$ by $2x+1$ we get $3x^2+5x-2$

i.e., $q(x)= 3x^2+5x-2$ and $r(x)= 0$

Sol.2 $\because, \alpha, \beta$ **are zeros of $2y^2+7y+5$**

$\therefore, \alpha + \beta = \frac{-7}{2}$ $\alpha\beta = \frac{5}{2}$

Now $(\alpha+\beta) + \alpha\beta = \frac{-7}{2} + \frac{5}{2}$

$= \frac{-2}{2} = -1$

$\therefore, \alpha + \beta + \alpha\beta = -1$

Sol. 3 $p(x) = 5x^2-4-8x$

$= 5x^2-8x-4$

$=5x^2-10x+2x-4$

$=5x(x-2)+2(x-2)$

$=(5x+2)(x-2)$

Zeros of quadratic polynomial are 2 and $\frac{-2}{5}$

Sum of zeroes $= 2+\left(\frac{-2}{5}\right) = \frac{10-2}{5} = \frac{8}{5} = -\left(\frac{-8}{5}\right) = \frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros $= 2 * -\frac{2}{5} = \frac{-4}{5} = \frac{c}{a} = \frac{constant\ term}{coefficient\ of\ x^2}$

Q4. $\because \alpha, \beta$ *are the zeros of the polynomials* $x^2 - px + q$

$\therefore\ \alpha + \beta = \frac{-b}{a} = \frac{p}{1} = p$

$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$

Value of (a) $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$=(p)^2 - 2 * q = p^2 - 2q$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{p}{q}$

$\therefore\ \alpha^2 + \beta^2 = p^2 - 2q\ \ and\ \frac{1}{\alpha} + \frac{1}{\beta} = \frac{p}{q}$

Q5. $p(x) = x^3 + 2x^2 - 5x - 6$

$q(x) = x + 1$

$r(x) = -4x - 4.\ \ Find\ g(x)$

$p(x) = g(x) * q(x) + r(x)$

$x^3 + 2x^2 - 5x - 6 = g(x) * (x + 1) + (-4x - 4)$

Or $\frac{x^3+2x^2-5x-6+4x+4}{x+1} = g(x)$

Or $\frac{x^3+2x^2-x-2}{x+1} = g(x)$

Dividing we get,

$g(x) = x^2 + x - 2$

Q6: $\because (x + a) is\ a\ factor\ of\ p(x) = 2x^2 + 2ax + 5x + 10$

$\therefore -a\ is\ the\ zero\ of\ the\ polynomial.$

$\therefore p(-a) = 0$

$2a^2 - 2a^2 - 5a + 10 = 0$

$-5a = -10$

$a = \frac{10}{5} = 2$

LEVEL III

Q1 Since two zeros of the given polynomial are $2 + \sqrt{3}$ & $2 - \sqrt{3}$

$\therefore [x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$ is a factor of the given polynomial

$\therefore [(x - 2)^2 - (\sqrt{3})^2]$ is a factor of polynomials

$x^2 + 4 + 4x - 3 = (x^2 - 4x + 1) is\ a\ factor\ of\ polynomial$

$\therefore 2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(2x^2 - x - 1)$

The other two zeros will be obtained by factorizing $2x^2 - x - 1$

$2x^2 - x - 1 = 2x^2 - 2x + x - 1$

$= 2x(x - 1) + 1(x - 1)$

$= (2x + 1)(x - 1)$

$Its\ zeros\ x = \frac{-1}{2}, 1$

Q2. Since α & β $are\ the\ zeros\ of\ the\ polynomials\ f(x) = 3x^2 - 4x + 1$

$$\alpha + \beta = -\left(-\frac{4}{3}\right) \qquad \& \qquad \alpha\beta = \frac{1}{3}$$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - \alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{4}{3}\right)^3 - 3*\frac{4}{3}*\frac{1}{3}}{\frac{1}{3}} = \frac{28}{9}$$

$$\frac{\alpha^2}{\beta} * \frac{\beta^2}{\alpha} = \alpha\beta = \frac{1}{3}$$

Hence the required polynomial $g(x) = k(x^2 = sx + p)$

$= k(x^2 - \frac{28}{9}x + \frac{1}{3})$ Where k is any non –zero real number.

Q3. $f(x) = x^2 - 8x + k$

Let α, β $the\ roots\ of\ f(x)$

$$\alpha + \beta = 8, \alpha\beta = k$$

$$\alpha^2 + \beta^2 = 40$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow (8)^2 - 2k = 40$$

$$\Rightarrow 64 - 2k = 40$$

$$\Rightarrow -2k = -24$$

$$\Rightarrow k = 12$$

$Q4:$ $Dividing\ 8x^4 + 14x^3 - 2x^2 + 7x - 8\ by\ 4x^2 + 3x - 2$

$we\ get$

$$4x^2 + 3x - 2)8x^4 + 14x^3 - 2x^2 + 7x - 8(2x^2 + 2x - 1$$

$$-8x^4 \pm 6x^3 \mp 4x^2$$

$$8x^3 + 2x^2 + 7x - 8$$

$$-8x^3 \pm 6x^2 \mp 4x$$

$$-4x^2 + 11x - 8$$

$$\mp 4x^2 \mp 3x \pm 2$$

$$14x - 10$$

$q(x) = 2x^2 + 2x - 1$ & $r(x) = 14x - 10$

Thus if we subtract the remainder $14x - 10$ *from* $8x^4 + 14x^3 - 2x^2 + 7x - 8$, *it will be divisible by* $4x^2 + 3x - 2$

Q5: *If* $x^4 + x^3 + 8x^2 ax + b$ *is exactly divisible by* $x^2 + 1$, *then remainder should be zero*

On dividing we get

$$x^2 + 1)x^4 + x^3 + 8x^2 + ax + b(x^2 + x + 7$$

$$-x^4 \quad \pm x^2$$

$$x^3 + 7x^2 + ax + b$$

$$-x^3 \quad \pm x$$

$$7x^2 + (a-1)x + b$$

$$-7x^2 \quad \pm 7$$

$$x(a-1) + b - 7$$

Now Remainder =0

$x(a-1) + b - 7 = 0$

$\Rightarrow x(a-1) + (b-7) = 0.x + 0$

$\Rightarrow x(a-1) = 0$ & $(b-7) = 0$

$\Rightarrow a = 1$ & $b = 7$

LINEAR EQUATION IN TWO VARIABLES

LEVEL 1

Q1 Let the number of boys=x

& number of girls=y

according to question

x+y=10 ---------------------(i)

x+y=4------------------------(ii)

for eq (i)

x	5	6	4
y	5	4	6

For eq (ii)

x	1	2	-1
y	5	6	3

Q2 Given equations are

x+2y=1

x-2y=-7

for eq(1)

X	-1	1	-3
y	1	0	2

For eq (2)

x	-3	-1	1
y	2	3	4

The lines intersect at point (-3,2) & hence solution of given pair of equations are (-3,2).

Q3 Given given pair of equations are

3x-y=3

2x+y=5

$a_1/a_2=3/2$, $b_1/b_2=-1/1$, $c_1/c_2=3/5$

$a_1/a_2 \neq b_1/b_2 \neq c_1/c_2$

Therefore given pair of equations have unique solution & are consistent.

Q4 Given given pair of equations are

x-2y=4-------------(i)

x-y=3----------------(ii)

from eq (i)

x=4+2y

substituting in eq(ii)

y=-1

& x=4-2=2

Q5 Given given pair of equations are

3x +2y=12------------(i)

5x-2y=4-------------(ii)

adding equation (i)&(ii)

x=2

substituting x=2 in eq (ii)

y=3

Hence x=2 & y=3

LEVEL 2

Q1 Given given pair of equations are

2x+3y=10-----------------(i)

4x+6y=12-----------(ii)

for eq (i)

x	5	2	-1
y	0	2	4

For eq (ii)

x	3	0	-3
y	0	2	4

These lines are parallel to each other and hence Given given pair of equations have no solution.

Q2 Given pair of equations are

5x+2y=k

10x+4y=3

for infinitely many solutions

$a_1/a_2 = b_1/b_2 = c_1/c_2$

i.e. 5/10=2/4=k/3

Hence k=3/2

Q3 Let unit digit of two digit number is=x

N & ten's digit=y

So two digit number will be=10y+x

Number formed by reversing the digits will be=10x+y

According to question

10y+x+10x+y=165

11x+11y=165----------------(i)

& x-y=3---------------(ii)

Multiplying eq(ii) by 11 we get

11x-11y=33------------- (iii)

Adding eq(ii)&(iii)

x=9

Substituting value of x in eq(ii) we get

y=6

Therefore number will be =6(10) +9=69

Q4 We have <A=x, <B=3x, <C=y

Given condition is

3y-5x=30----------- (i)

By angle sum property of triangle

<A+<B+<C=180^0

x+3x+y=180^0

4x+y=180^0

From eq (ii)

y=180^0-4x

Substituting in eq(i) we get

x=30

Therefore <A=30^0

<B=3(30) =90^0

& <C=180-4(30) =60^0

Q5 Given pair of equations is

5x+3y=5xy

2x+4y=3xy

Dividing both sides by xy we get

5/y + 3/x = 5 -----------------(1)

And 2/y + 4/x = 3 ----------------- (2)

let 1/x = u and 1/y = v we get ,

3u +5v = 5 ---------------------(3)

4u +2v = 3 ---------------------(4)

solving eq (3) and (4) by elimination we get ,

u = 5/14 and v = 11/14

now 1/x=5/14 & 1/y=11/14

Therefore x= 14/5 & y = 14/11

LEVEL 3

Q1 Given pair of lnear equation are

$4x - 3y = -4$

$4x + 3y = 20$

for eq (1)

x	-1	2	-4
y	0	4	-4

For eq (2)

x	5	2	8
y	0	4	-4

Area of triangle formed by these lines and x-axis = 1/2×b ×h

= 1/2×6×4

=12 sq. units

Q2 Given pair of linear equation are

$a^2 /x - b^2 /y = 0$

$a^2 b /x + b^2 a /y = a+b$, $x,y \neq 0$

let 1/x = u and 1/y = v

now eq becomes ,

$a^2 u - b^2 v = 0$ ----------------------------(1)

$a^2 bu + b^2 av = a+b$ ----------------------------(2)

by cross multiplication we get,

$a^2 /x - b^2 /y = 0$

$u/b^2 (a+b) = -v/- a^2 (a+b) = 1/a^3 b^2 + a^2 b^3$

$u = 1/a^2$ and $v = 1/ b^2$

now we have

1/x = u and 1/y = v

$x=a^2$ and $y = b^2$

Q3 Let speed of X =x km/hr

& speed of Y= ykm/hr

distance traveled = 30km

time taken by X = 30/x hr

time taken by Y = 30/y hr

A.T.Q.

$30/x - 30/y = 3$------------------(1)

$30/y - 30/2x = 3/2$------------------(2)

let $1/x = p$ and $1/y = q$ we will get ,

$30p - 30q = 3$ --------------------(3)

$-15p + 30q = 3/2$ -------------------(4)

adding (3) and (4) we get,

$p = 3/10$ and $q = 1/5$

we have,

$1/x = 3/10$ and $1/y = 1/5$

Hence ,

$x = 10/3$ km/hr and $y = 5$km/hr

Q.N.4 Let amount invested at 12% p.a. = x

Let amount invested at 10% p.a. = y

A.T.Q

$12x/100+10y/100=130$

$6x+5y=6500$--------------------(1)

and

$12y/100 + 10x/100 = 134$

$5x + 6y = 6700$ ----------------(2)

solving (1) and (2) we get x= Rs 500

y = Rs 700

Q5 Let speed of boat in still water = xkm/hr

And speed of stream = y km/hr

speed of boat upstream = (x-y) km/hr

speed of boat downstream = (x+y) km/hr

A.T.Q,

32/x-y + 36/x+y = 7----------------(1)

40/x-y + 48/x+y = 9 --------------(2)

let 1/x-y = p and 1/x+y = q we get,

32p +36q = 7---------------(3)

40p + 48q = 9---------------(4)

solving eq (3) and (4) we get,

p = 1/8 and q= 1/12

also 1/x-y = p and 1/x+y = q

so x-y =8 ----------------(5)

x+y=12---------------(6)

solving eq (5) &(6) we get

x= 10 and y= 2

therefore

speed of boat in still water = 10km/hr

And speed of stream = 2 km/hr

<u>QUADRATIC EQUATIONS</u>

<u>LEVEL I</u>

<u>Q1</u>.

For Equal Roots

Discriminant =0

B^2-4ac =0

K^2-4x2x8 =0

K^2-64=0

$(K+8)(k-8) = 0$

$K = 8, -8$

Q2

$x^2 - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x-5)(x+2) = 0$

$X = 5, -2$

Q3.

$3x^2 - 3x - 2x + 2 = 0$

$3x(x-1) - 2(x-1) = 0$

$(x-1)(3x-2) = 0$

$x = 1, 2/3$

Q4.

$x - 1/x = 4$

X2-1 = 4x

X2-4x-1 =0

D = b2- 4ac

= (-4)2 – 4x1x -1

= 16 + 4

=20

Roots are real and unequal.

$X = \frac{-b \pm \sqrt{d}}{2a}$

$= \frac{-(-4) \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2}$

$= \frac{2(2 \pm \sqrt{5})}{2}$

$= (2 \pm \sqrt{5})$

Q5.

4 is the root of the given equation.

Then $(4)^2 - 5(4) + K = 0$

$16-20 + k = 0$

$-4 + k = 0$

$K = 4$

Q6.

$a= 3, \quad b = -5 \quad c= 2$

$D = b^2- 4ac$

$= (-5)^2- 4 \times 3 \times 2$

$= 25 - 24$

$= 1$

D= +ve

Therefore roots are real and unequal.

Level –II

Q1

For equal roots

$D = 0$

$b^2- 4ac = 0$

$(4\alpha-12)^2 -4 (\alpha -3)(4) =0$

$16 \alpha^2 + 144- 96 \alpha - 16 \alpha +48 = 0$

$16 \alpha^2 - 112 \alpha + 192 = 0$

Or $\alpha^2 - 7 \alpha + 12 = 0$

$(\alpha-3) (\alpha-4) = 0$

$\alpha = 4$ as $\alpha = 3$ is rejected

Q2. $9x^2 - 3ax-3bx + ab = 0$

$3x(3x-a) - b (3x-a) = 0$

$(3x-a) (3x-b) = 0$

$X = a/3, \ b/3$

Q3. $6x^2 -13x= 5$

Multiply both sides by 6

$36x^2 - 78x = 30$

$(6x)^2 - 2.6x.13/2 + \left(\frac{13}{2}\right)^2 = 30 + \left(\frac{13}{2}\right)^2$

$(6x - \frac{13}{2})^2 = 30 + \frac{169}{4}$

$(6x - \frac{13}{2})^2 = \frac{289}{4}$

$(6x - \frac{13}{2}) = \pm\frac{17}{2}$

$(6x - \frac{13}{2}) = \frac{17}{2}$ 0r $(6x - \frac{13}{2}) = -\frac{17}{2}$

$6x = \frac{13}{2} + \frac{17}{2}$ 0r $6x = \frac{13}{2} - \frac{17}{2},$

$6x = \frac{30}{2}$ 0r $6x = -\frac{4}{2},$

$x = \frac{5}{2}$ 0r $x = \frac{-1}{3},$

Q4. Let first number be x

Second number be x + 5

According to the statement

$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10}$

$$\frac{x+5-x}{x(x+5)} = \frac{1}{10}$$

$$\frac{5}{x^2+5x} = \frac{1}{10}$$

$$x^2 + 5x = 50$$

$$x^2 + 5x - 50 = 0$$

$$x^2 + 10x - 5x - 50 = 0$$

X (x + 10) -5 (x + 10) = 0

(x + 10) (x - 5) = 0

X = -10 , 5

If first number is -10 then second number is -5

If first number is 5 then second number is 10

Q5.

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$

$\frac{(x-4)(x-1)+(x-2)(x-3)}{(x-2)(x-4)}=\frac{10}{3}$

$\frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$

Or

$$\frac{x^2-5x+5}{x^2-6x+8}=\frac{5}{3}$$

Simplifying

$3(x^2-5x+5) = 5(x^2-6x+8)$

$3x^2$- 15x + 15 = $5x^2$ -30x + 40

$2x^2$ -15x + 25 = 0

$2x^2$ -10x -5x + 25 = 0

2x (x- 5) -5 (x-5) = 0

(2x -5) (x-5) = 0

X= 5/2 or 5

LEVEL –III

Q1.

Put $\frac{2x-1}{x+3}=y$ then

2y – 3/y = 5

$$\frac{2y^2-3}{y}=5$$

$2y^2-3=5y$

$$2y^2-5y-3=0$$

$$2y^2-6y+y-3=0$$

2y (y-3) + 1 (y- 3) = 0

$(2y+1)(y-3) = 0$

$Y = -1/2$ or $y = 3$

Either $\frac{2x-1}{x+3} = 3$

$2x - 1 = 3x + 9$

$X = -10$

Or $\frac{2x-1}{x+3} = -\frac{1}{2}$

$4x - 2 = -x - 3$

$5X = -1$

$X = -1/5$

Q2.

Let the side of the smaller square is x cm

And the side of the bigger square is y cm.

Then $y^2 - 2x^2 = 14$

Or $y^2 = 2x^2 + 14$ ....(i)

Also

$2y^2 + 3x^2 = 203$

From Equation (i)

$2(2x^2 + 14) + 3x^2 = 203$

Or $7x^2 = 203 - 28$

Or $7x^2 = 175$

Or $x^2 = 25$

Or $x = 5$

From Equation (i)

$y^2 = 2(5)^2 + 14$

$= 50 + 14$

$= 64$

$Y = 8$ cm

the side of the smaller square is 5 cm

And the side of the bigger square is 8 cm.

Q3.

X= $\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\cdots\ldots..}}}}$

X = $\sqrt{20+x}$

Squaring both sides

x^2 -x- 20 = 0

x^2 -5x + 4x -20 = 0

X (x-5) + 4 (x-5) = 0

(x-5)(x-4) = 0

X = 5 , -4

Q4.

Let the age of Father is x years

And the age of the son is y Years

According to the statement

X + y = 45

Or y = 45 – x ....(i)

Also (x-5)(y-5) = 124

Using equation (i)

(x-5)(45-x-5) = 124

(x-5)(40-x) = 124

$$-x^2 + 45x - 200 = 124$$

$$x^2 - 45x + 324 = 0$$

$$x^2 - 36x - 9x + 324 = 0$$

X(x-36) -9 (x- 36) = 0

(x-9)(x-36) = 0

X = 9,36

Let the age of Father is 36 years

And the age of the son is y= (45 – 36) Years

= 9 Years

Q 5.

Total Distance to be covered = 600 km

Let normal average speed of the plane = x kmph

Reduced speed of the plane due to bad weather= (x-200) kmph

Time taken by the plane in normal conditions =$\frac{600}{x}$

Time Taken by Plane in bad weather = $\frac{600}{x-200}$

According to the statement

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$

$$\frac{600x - 600x + 120000}{x(x-200)} = \frac{1}{2}$$

$$\frac{120000}{x^2 - 200x} = \frac{1}{2}$$

$$x^2 - 200x = 240000$$

$$x^2 - 200x - 240000 = 0$$

$$x^2 - 600x + 400x - 240000 = 0$$

X(x-600)+400(x-600) = 0

(x-600)(x+400)= 0

X = 600, -400

Speed of the Air craft in normal condition is 600 kmph

Time taken by the aircraft to cover the 600km is 600/600 = 1 Hour.

ARITHMETIC PROGRESSION

LEVEL1

Q1 Given A.P. is 51,59,67,75,.....................

Here a = 51 , d = 59-51 = 8

Next two terms are 75 + 8 = 83

And 83 + 8 = 91

Q2 Let a be the first term and d be the common difference of A.P.

$T_3 = 5$

a + 2d = 5 -----------------(1)

and $T_7 = 9$

a+6d = 9 -------------------- (2)

solving (1) and (2) we get

$d = 1$

from eq (1) $a + 2(1) = 5$

so $a = 5-2= 3$

Hence A.P. IS 3,4,5,6,.................................

Q3 Given A.P. is 5,11,17,.............................299

Here $a = 5$ and $d= 11-5= 6$

Let $a_n = 299$

$a + (n-1) d = 299$

$5 + (n-1) 6 = 299$

$6n-6=294$

$6n=294+6=300$

$n=300/6=50$

Now $T_{16} = a+15d$

$=5+15(6)=95$

Q NO.4 Given A.P is 3,8,13...................,253

A.P in reverse order is 253,..............,13,8,3

Here $a=253$, $d=8-13=-5$

$T_{20} = a+19d =253+19(-5) =253-95 = 158$

Q NO.5 Given A.P is 5,8,11,.....................,320.

Herea=5 , d=8-3=5

Let $a_n=320$

$5+(n-1)3=320$

$3n-3=320-5=315$

$3n=315+3=318$

$n=318/3=106$

Hence 320 is 106^{th} term.

Q NO.6 Let $T_n =5n-3$

$T1=5(1)-3=2$

$T2=5(2)-3=10-3=7$

$T3=5(3)=15-3=12$

A.P.i s

2,7,12,................

T_1=3+4(1)=7

T_2=3+4(2)=11

T_3=3+4(3)=15

A.P.is 7,11,15,.....................................

Here a=7 ,d=11-4=7 ,n=15

S_{15}=15/2(14+56)=15/2(70)=525

LEVEL 2

Q NO 1 Given A.Pis 7, 10,13..........................

Here a=7, d=3

Let a_n= 68

a+(n-1)d=68

7+(n-1)3=68

3n-3=61

3n=64

n=64/3 which is not a whole number. Therefore, 68 is not a term of given A P

Q NO 2 To show that a-b, a, a+b are consecutive terms of A.P

HereT_1=a-b, T_2=a, T_3=a+b

T_2-T_1=a-(a-b)=a-a-b=ba+2d= 5

T_3-T_2=B

T_2-T_1=T_3-T_2=b

Hence given terms are consecutive terms of the A.P.

QNO3 Given A.P. is 3,10,17,..............................

Here a = 3 , d = 10 -3 =7

Let a_n be the term that is 84 more than its 13[th] term

∴ a + (n-1) d = a + 12d =84

(n-1) 7 = 12×7+84

n =25

hence required term is 25[th] term

QNO4 Let a be the first term and d be the common difference

$S_n = \frac{5n^2}{2} + \frac{3n}{2}$

$S_{20} = \frac{5(20)^2 + 3(20)}{2}$

S_{20} = 931

∴ $a_n = S_{20} - S_{19} = 1030 - 931 = 99$

QNO5 Let a be the first term and d be the common difference

$T_8 = 37$

$a + 7d = 37$--------------------(1)

$T_{15} = 15 + T_{12}$

$a + 14d = 15 + a + 11d$

$14d - 11d = 15$

$3d = 15$

∴ $d = 5$

Now from eq (1)

$a + 7(5) = 37$

$a = 2$

hence A.P. is 2 ,7 ,12 ,------------------

$S_{15} = \frac{15}{2}[2X\,2 + (15-1)5]$

$= \frac{15}{2}[4 + 70]$

$= \frac{15}{2}[74]$

$= 555$

QNO 6 $S_n = 3n^2 - 4n$

$S_{n-1} = 3(n-1)^2 - 4(n-1)$

$= 3[n^2 + 1 - 2n] - 4n + 4$

$= 3n^2 + 3 - 6n - 4n + 4$

$= 3n^2 - 10n + 7$

∴ $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7) = 6n - 7$

LEVEL3

QNO1 Let a be the first term and d be the common difference

Given a =2 , d=8

$S_n = 90$

$\frac{n}{2}\ [2a + (n-1)d] = 90$

$n[2X2 + (n-1)8] = 90X2$

$n[4 + 8n - 8] = 180$

$8n^2 - 4n - 180 = 0$

$2n^2-n-45 = 0$

$2n^2-10n+9n -45 = 0$

$2n(n-5) + 9(n-5) = 0$

$(n-5)(2n-9) = 0$

$\therefore$ n=5 , $n = \frac{-9}{2}$ (rejected)

Now $a_n = a + (n-1)d$

$= 2 + (5-1)\,8$

$= 34$

QNO2 Given A.P. is 9 ,17, 25,------------------

Here a=9 and d = 17-9 =8

$S_n = 636$

$\frac{n}{2}[2a + (n-1)d] = 636$

$n[2X9 + (n-1)8] = 636x2$

$n[18 + 8n - 8] = 1272$

$8n^2-10n -1272 = 0$

$4n^2-5n -636 = 0$

$$n = \frac{-(5)\pm\sqrt{(5)^2-4x4x(-636)}}{2X4}$$

$$n = \frac{-(5)\pm\sqrt{25+10176}}{8}$$

$$n = \frac{-5\pm101}{8}$$

$$n = 12 \quad , \quad \frac{-106}{8} \text{ (reject)}$$

$\therefore$n=12 , hence 12^{th} term is required term

QNO3 As 3k+2 , 4k+3 , 6k-1 are in A. P.

$\therefore (4k+3) - (3k+2) = (6k-1) - (4k+3)$

$4k+3 -3k -2 = 6k-1-4k-3$

$K+1 = 2k-4$

$K -2k = -4-1$

$-k = -5$

$K=5$

QNO4 Let a be the first term , d be the common difference and l be the last term

here a = 17, l = 350 , d = 9

let nth term be the last term of the A. P.

$\therefore a_n = 350$

$a + (n-1)d = 350$

$17 + (n-1)9 = 350$

$17 + 9n - 9 = 350$

$9n = 342$

$n = 342/9$

$n = 38$

$\therefore S_{38} = \frac{38}{2}[17 + 350]$

$= 19 \times 367 = 6973$

QNO5 Given A.P. is 10,7,4, -------------------- -62

Here $a = 10$, $d = 7-10 = -3$

Let $a_n = -62$

$a + (n-1)d = -62$

$10 + (n-1)(-3) = -62$

$10 - 3n + 3 = -62$

$13 - 3n = -62$

$-3n = -62 - 13$

$-3n = - -75$

$n = 75/3 = 25$

$\therefore$Middle term = ($\frac{n+1}{2}$) th = $\frac{25+1}{2} th = 13^{th}$ term

Now $a_{13} = a + 12d = 10 + 12 \times (-3) = 10 - 36 = -26$

Triangles

LEVEL – 1

1. ΔABC ~ ΔDEF

$2AB = DE$

$\frac{AB}{DE} = \frac{1}{2}$

Also, $\frac{AB}{DE} = \frac{BC}{EF}$ SO, $\frac{1}{2} = \frac{8}{EF}$ So, EF = 16 cm

2.

$\frac{PS}{SQ} = \frac{PT}{TR}$ ----------------------------(Given)

By converse of Thale's Theorem

ST || QR

So, ∠ PST = ∠ PQR -------------(corresponding angles)

But, ∠ PST = ∠ PRQ

So, ∠ PQR = ∠ PRQ

So, PQ = PR -------------(sides opposite to equal angles)

Hence, ΔPQR is an isosceles triangle.

3.

$\Delta ADE \sim \Delta ABC$ ------------------(given)

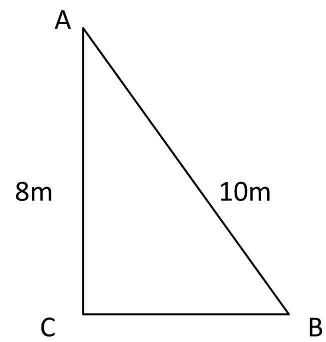
$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$\frac{25}{ar(\Delta ABC)} = \frac{4^2}{8^2}$

$\frac{25}{ar(\Delta ABC)} = \frac{16}{64}$

ar(ΔABC) = 25 x 4 = 100 cm^2

4.



Length of ladder (AB)= 10m

Height of window (AC) = 8m

In Rt. ΔACB, By Pythagoras theorem

$AB^2 = BC^2 + AC^2$

$10^2 = BC^2 + 8^2$ So, $100 = BC^2 + 64$

$BC^2 = 36$ So, BC = 6 m

Hence, the distance between foot of the ladder and base of the wall is 6 m.

LEVEL – 2

Q1.

In ΔPMN, AB || MN
By Thale's Theorem
$\frac{PA}{PM} = \frac{PB}{PN}$

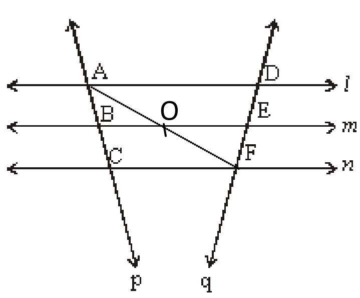
So, $\frac{x-2}{x} = \frac{x-1}{x+2}$

$x^2 - 2^2 = x^2 - x$
$-4 = -x$ so, x = 4

Q2.

Construction: - Join AF intersecting BE at pt. O

O

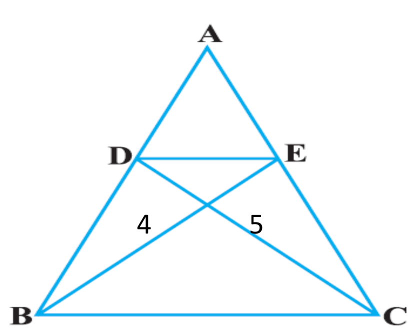


Proof: - In ΔACF, BO || CF (as m || n)
By Thale's theorem
$\frac{AB}{BC} = \frac{AO}{OF}$ --------(1)

Similarly, $\frac{DE}{EF} = \frac{AO}{OF}$ --------(2)
From (1) & (2), we get,
$\frac{AB}{BC} = \frac{DE}{EF}$

Hence, intercepts are proportional.

Q3.



In ΔABC, DE || BC -------(given)
By Thales Theorem
$\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{4}{9} = \frac{DE}{BC}$ --------------(1)

F

In ΔDEF & ΔCBF

$\angle EDF = \angle BCF$ ⎰ Alternate Interior
$\angle DEF = \angle CBF$ ⎱ Angles

By AA similarity,

ΔDEF ~ ΔCBF

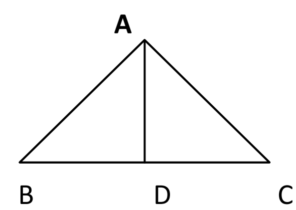
So, $\frac{ar(\Delta DEF)}{ar(\Delta BCF)} = \frac{DE^2}{BC^2} = \frac{4^2}{9^2} = \frac{16}{81}$

Hence Proved.

Q4.

$\frac{BD}{DA} = \frac{DA}{DC}$

So, $DA^2 = BD \times DC$



In ΔABC, AD is perpendicular to BC

In Rt. ΔABD, By Pythagoras theorem
$AB^2 = DA^2 + BD^2$ ---------(1)
In Rt. ΔACD, By Pythagoras theorem
$AC^2 = DA^2 + CD^2$ ---------(2)

Adding (1) and (2) we get,

$AB^2 + AC^2 = DA^2 + BD^2 + DA^2 + CD^2$

$AB^2 + AC^2 = 2DA^2 + BD^2 + CD^2$

$AB^2 + AC^2 = 2\ BD \times DC + BD^2 + CD^2$

$AB^2 + AC^2 = (BD + CD)^2$

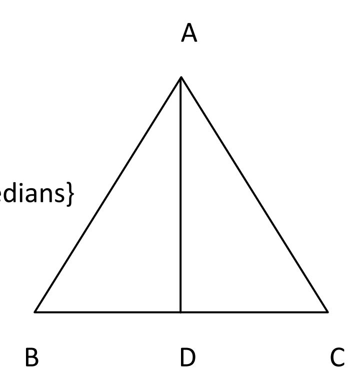
$AB^2 + AC^2 = BC^2$

By converse of Pythagoras theorem, ΔABC is a right angled triangle right angled at A

LEVEL – 3

Q1. Refer theorem 6.6 of NCERT textbook (Class X)

Q2. In ΔABD and ΔPQM



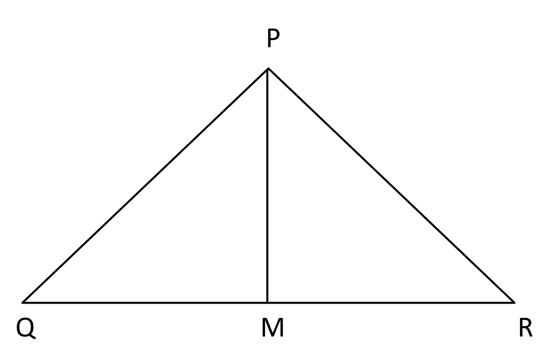
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

So, $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$ {AD & PM are medians}

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

So, ΔABD ~ ΔPQM (SSS Similarity)

So, $\angle B = \angle Q$



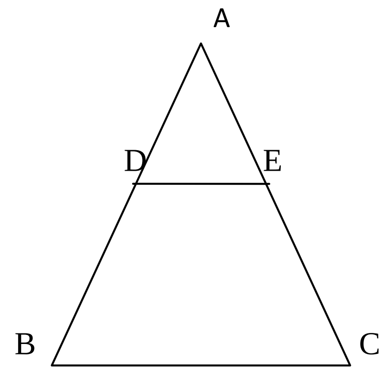
In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$

So, ΔABC ~ ΔPQR (SAS Similarity)

Hence Proved

Q3. In ΔABC



DE|| BC (given)

$\Rightarrow \angle D = \angle B \quad alternate \angle's$

$\Rightarrow \angle E = \angle C \quad alternate \angle's$

$\Rightarrow In$ ΔADE & ΔABC

$\angle D = \angle B$

$\angle E = \angle C$

By AAA Similarity Criteria

$\Delta ADE \sim \Delta ABC$

Also $\frac{\text{area } (\Delta ADE)}{\text{area } (\Delta ABC)} = \frac{AD^2}{AB^2}$

$\Rightarrow \frac{AD^2}{AB^2} = \frac{1}{3}$

$\Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{3}}$

Subtracting both sides from '1'

$\Rightarrow 1 - \frac{AD}{AB} = 1 - \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{AB - AD}{AB} = \frac{\sqrt{3} - 1}{\sqrt{3}}$

$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{3} - 1}{\sqrt{3}}$

Q4: See the NCERT BOOK for the theorem.

Q5: See the NCERT BOOK for the theorem.

Q6: See the NCERT BOOK for the theorem.

COORDINATE GEOMETRY

LEVEL 1

Sol. 1 Distance PQ = $\sqrt{(x_{2-}x_1)2 + (y_2 - \ y_1)2}$

$= \sqrt{(2-4)^2 + (-3-4)^2}$

$= \sqrt{53}$

<u>Sol.2.</u> **AB =** $\sqrt{[2-(-2)]^2+\ [1-(-1)]^2}$

= $\sqrt{16+4}$

= $2\sqrt{5}$

BC = $\sqrt{(4-2)^2+(2-1)^2}$

= $\sqrt{5}$

AC = $\sqrt{[4-(-2)]^2\ +[2-(-1)]^2}$

= $\sqrt{36+9}$

= $3\sqrt{5}$

Since, AB+BC = $2\sqrt{5}+\sqrt{5}$

= $3\sqrt{5}$

= AC

Hence , given points are collinear.

<u>Sol.3</u> **x=** $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

= $\frac{2\times(-3)+3\times7}{2+3}$

= $\frac{-6+21}{6}$

= 3

Y= $\frac{m_1y_2+m_2y_1}{m_1+m_2}$

= $\frac{2\times4+3\times(-1)}{2+3}$

= $\frac{8-5}{5}$

= 1

∴, the reqd. coordinates of a point are (3,1)

<u>Sol. 4</u> **x=** $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

-2 = $\frac{4m_1-3m_2}{m_1+m_2}$

-$2m_1 - 2m_2 = 4m_1 - 3m_2$

$m_2 = 6m_1$

Or $\frac{m_1}{m_2}$ = $\frac{1}{6}$

Thus required ratio is 1:6

Sol. 5 Area of triangle= $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}[1(5-7) + 3(7+1) + 2(-1-5)]$

=$\frac{1}{2}$[10]=5 square units

LEVEL – II

Sol. 1 AB =$\sqrt{(x_2 - x_1)2 + (y_2 - y_1)2}$

= $\sqrt{(6-0)^2 + (3-5)^2}$

=$\sqrt{36+4}$

=$\sqrt{40}$

BC =$\sqrt{(5-6)^2 + (10-3)^2}$

=$\sqrt{1+49}$

=$\sqrt{50}$

AC = $\sqrt{(5-0)^2 + (10-5)^2}$

=$\sqrt{25+25}$

=$\sqrt{50}$

Since ,BC = AC$\neq$ AB

∴, These are the vertices of an isosceles triangle

Sol.2 Let the reqd. pt. be =A (x,0)

$\sqrt{(7-x)^2 + (6-0)^2}$ = $\sqrt{(9-x)^2 + (4-0)^2}$

$\sqrt{49 - 14x + x^2 + 36}$ = $\sqrt{81 - 18x + x^2 + 16}$

$85-14x+x^2= 97-18x+x^2$

4x =12

X = 3

Thus reqd. pt. is (3,0)

Sol. 3 Y= $\frac{m_1 y_2+m_2 y_1}{m_1+m_2}$

0 =$\frac{4\,m_1+(-7)m_2}{m_1+m_2}$

Or 0 = $4m_1 - 7m_2$

Or $4\,m_1 = 7m_2$

Or $\frac{m_1}{m_2} = \frac{7}{4}$

Thus required ratio is 7:4

Now, x= $\frac{m_1 x_2+m_2 x_1}{m_1+m_2}$

=$\frac{7\times 6+4\times 1}{7+4}$

=$\frac{46}{11}$

∴, The required point is($\frac{46}{11}$,0)

Sol.4 x= $\frac{4+0}{2}$

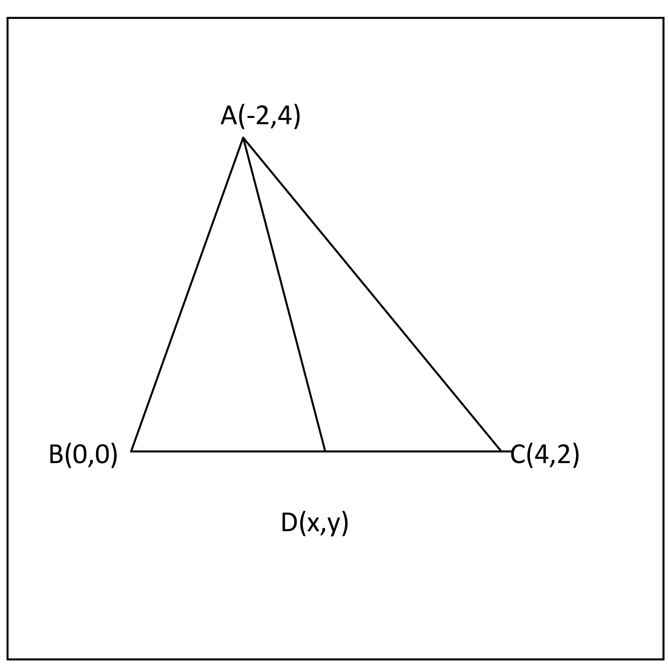
= 2

Y = $\frac{2+0}{2}$

= 1

AD = $\sqrt{(2+2)^2+(1-4)^2}$

= $\sqrt{16+9}$ = 5



Thus, length of the median is 5units.

Sol. 5 **Area of triangle = 0**

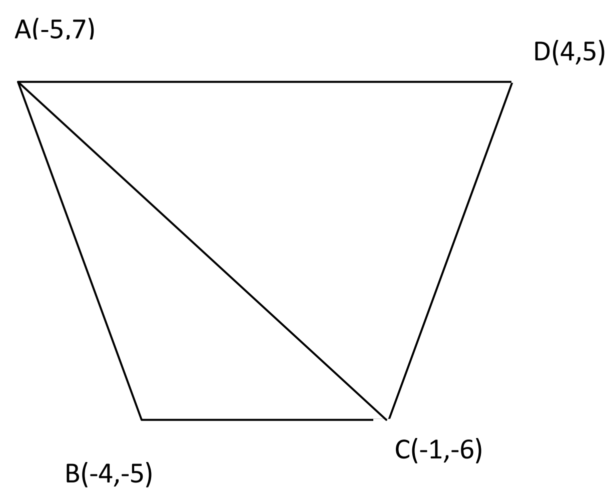
Or $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_{1-}y_2)]$**=0**

$\frac{1}{2}[1(k - 4) + 3(4 - 1) + (-1)(1 - k)]$ **= 0**

$\frac{1}{2}$**[2k+4]=0**

K= -2

Sol.6



Join diagonal AC of Quad. ABCD.

Area of ΔABC**=**$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_{1-}y_2)]$

=$\frac{1}{2}[-5(-5 + 6) - 4(-6 - 7) - 1(7 + 5)]$

=$\frac{1}{2}[-5 + 52 - 12]$

= 35/2 Sq Units.

Area of ΔACD**=**$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_{1-}y_2)]$

=$\frac{1}{2}[-5(-6 - 5) - 1(5 - 7) + 4(7 + 6)]$

=$\frac{1}{2}[55 + 2 + 52]$

= 109/2 Sq Units.

Area of Quad ABCD = Area of ΔABC + Area of ΔACD

=(35/2 + 109/2) Sq Units

= 72 Sq Units.

LEVEL – 3

Sol.1 LetA(0,0) , B(5,5$\sqrt{3}$) ,C(-5,5$\sqrt{3}$) are the vertices of the a triangle

AB= $\sqrt{(5-0)^2 + (5\sqrt{3}-0)^2}$

= $\sqrt{25+75}$

= 10

BC = $\sqrt{(5+5)^2 + (5\sqrt{3}-5\sqrt{3})^2}$

= $\sqrt{100+0}$

= 10

AC = $\sqrt{(-5-0)^2 + (5\sqrt{3}-0)^2}$

=$\sqrt{25+75}$

= 10

AB = BC= AC = 10

Therefore the given points are the vertices of an equilateral triangle.

Sol. 2.

XY= $\sqrt{(3/2-0)^2 + (-2+4)^2}$

XY = $\sqrt{(3/2)^2 + (2)^2}$

XY = $\sqrt{\frac{9}{4} + 4}$

= 5/2

YZ= $\sqrt{(3-3/2\,)^2+\ (0+2\,)^2}$

= $\sqrt{(3/2\,)^2+\ (2\,)^2}$

YZ = $\sqrt{\frac{9}{4}+\ 4}$

= 5/2

XZ= $\sqrt{(3-0\,)^2+\ (0+4\,)^2}$

= $\sqrt{9+16}$

= 5

XZ^2 = 5^2 = 25

$XY^2+\ YZ^2$= $(5/2)^2$ + $(5/2)^2$

= 50/4

= 25/2

$XY^2+\ YZ^2\neq XZ^2$

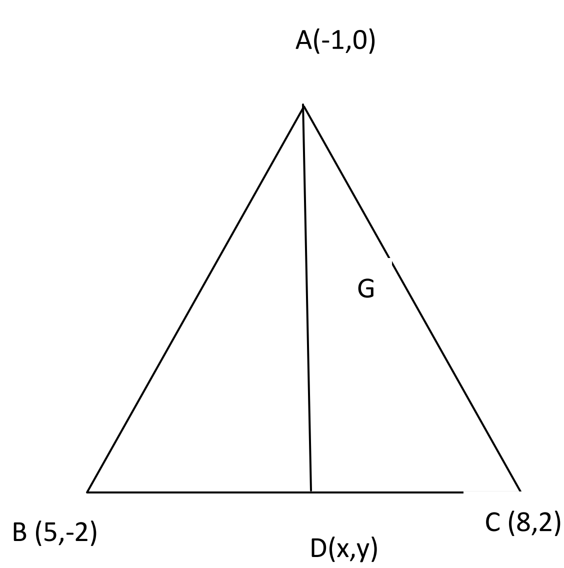
Therefore it is not a right triangle…………(i)

Y = YZ……………… Proved,

Therefore it is an Isosceles Triangle…………………..(ii)

From (i) and (ii) XYZ is not an Isosceles right triangle.

Sol.3.



X = $\frac{5+8}{2}$

X = $\frac{13}{2}$

y = $\frac{-2+2}{2}$

y=0

Centroid G divides the median AD in the ratio of 2 : 1

Let coordinates of G are (a, b)

Applying section formula

a $= \dfrac{\frac{13}{2}\times 2+1(-1)}{2+1}$

$= \dfrac{13-1}{3}$

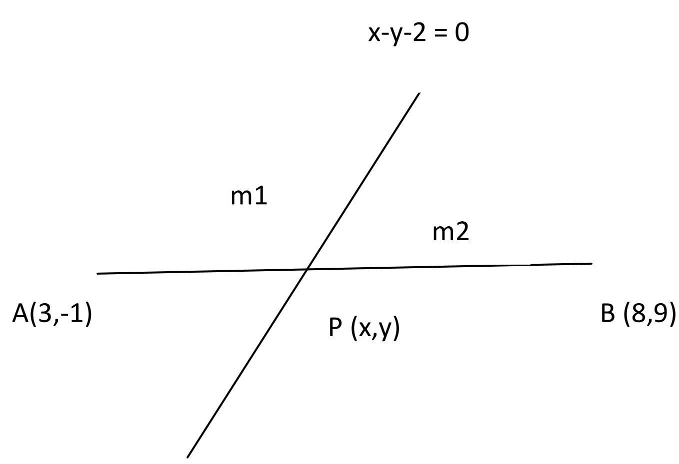
= 4

b $= \dfrac{2\times 0+1(0)}{2+1}$

b =0

Thus the coordinates of centroid are (4,0).

<u>Sol.4</u>.



, **x=** $\dfrac{m_1x_2+m_2x_1}{m_1+m_2}$

, **y=** $\dfrac{m_1y_2+m_2y_1}{m_1+m_2}$

X = $\dfrac{8m_1+3m_2}{m_1+m_2}$ **,** **y =** $\dfrac{9m_1+(-1)m_2}{m_1+m_2}$

Put the values of x and Y in x-y-2=0

$$\frac{8m_1 + 3m_2}{m_1 + m_2} - \frac{9m_1 + (-1)m_2}{m_1 + m_2} - 2 = 0$$

$\frac{8m_1 + 3m_2 - 9m_1 + m_2 \; - 2(m_1 + m_2)}{m_1 + m_2} = 0$,

$$-3m_1 + 2m_2 \;\; = 0$$

$$3m_1 = 2m_2$$

$$\frac{m_1}{m_2} = \frac{2}{3}$$

Thus the required ratio is 2: 3

x= $\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$, **y=** $\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

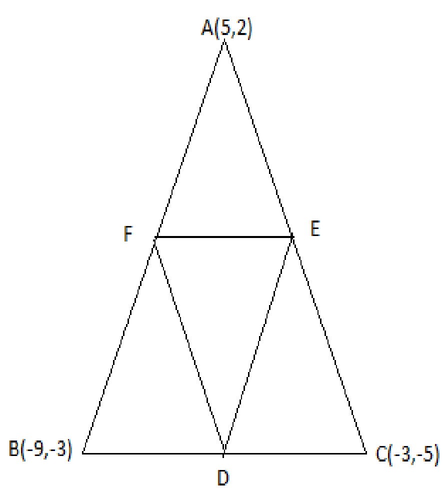
X = $\frac{8(2)+3(3)}{2+3}$, **y =** $\frac{9(2)+(-1)3}{2+3}$

X = $\frac{25}{5}$, **y =** $\frac{15}{5}$

X = 5 and Y=3

Thus the coordinates of the point of intersection is (5,3)

<u>Sol. 5</u>



D is the mid point of BC

∴Co-ordinates of D are $\left[\frac{-9-3}{2}, \frac{-3-5}{2}\right]$

ie. $[-6, -4]$

E is the mid point of AC

∴Co-ordinates of E are $\left[\frac{5-3}{2}, \frac{2-5}{2}\right]$

ie. $\left[1, \frac{-3}{2}\right]$

F is the mid point of AB

∴Co-ordinates of F are $\left[\frac{5-9}{2}, \frac{2-3}{2}\right]$

ie. $\left[-2, \frac{-1}{2}\right]$

Area of ΔDEF= $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_{1-}y_2)]$

= $\frac{1}{2}\left[-6\left(-\frac{3}{2}+\frac{1}{2}\right) + 1\left(-\frac{1}{2}+4\right) - 2(-4+\frac{3}{2})\right]$

= $\frac{1}{2}\left[-6(-1) + \frac{7}{2} + 5\right]$

= 29/4 Sq Units

Area of ΔABC= $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_{1-}y_2)]$

= $\frac{1}{2}[5(-3+5) + (-9)(-5-2) + (-3)(2+3)]$

= $\frac{1}{2}[10 + 63 - 15]$

= $\frac{1}{2}[58]$

=29 Sq Units

Now Area Of ΔDEF = $\frac{29}{4}$ Sq Units

Area of ΔABC = 29 Sq units

Area of ΔABC = 4 x AreaΔDEF

INTRODUCTION TO TRIGONOMETRY

LEVEL 1

Q 1 $\sin\theta = \frac{12}{13}$

P=12k H=13k

Base = $\sqrt{(13k)^2 - (12k)^2} = 5k$

$$\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{\frac{13k}{5k} + \frac{12k}{5k}}{\frac{13k}{5k} - \frac{12k}{5k}}$$

=25

Q 2 tanA=$\frac{1}{\sqrt{3}}$

P=1K B=$\sqrt{3}$ K H=

sinA=1/2 cosA=$\frac{\sqrt{3}}{2}$ tanA=$\frac{1}{\sqrt{3}}$ cotA=$\sqrt{3}$ secA=$\frac{2}{\sqrt{3}}$ cosecA=2

Q 3 sinθ=1/3

2cot$^2\theta$+2=2x (2$\sqrt{2}$)^2+2

=2x8+2

=18

Q4 $\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

$$= \frac{1}{\sqrt{2}} x \frac{1}{2} + \frac{1}{\sqrt{2}} x \frac{\sqrt{3}}{2}$$

$$= = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

Q5

$$\cos ec 30^\circ + \cot 45^\circ$$
$$= 2 + 1$$
$$= 3$$

Q6

$$2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$$
$$= 2x(\frac{1}{2})^2 x\sqrt{3} - 3x(\frac{1}{2})^2 x(\frac{2}{\sqrt{3}})^2$$
$$= \frac{\sqrt{3}}{2} - 1$$

Q7

$$\frac{\cos 37^\circ}{\sin 53^\circ}$$
$$=\frac{\cos\left(90-53\right)}{\sin 53^\circ}$$
$$=\frac{\sin 53^\circ}{\sin 53^\circ}$$
$$=1$$

Q8

$$\sin 39^\circ - \cos 51^\circ$$
$$=\sin(90-51)^\circ - \cos 51^\circ$$
$$=\cos 51^\circ - \cos 51^\circ$$
$$=0$$

Q9

$$\frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$=\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$
$$=\cos^2\theta-\sin^2\theta$$

Q10

$$\cos^4\theta-\sin^4\theta$$
$$=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$$
$$=\cos^2\theta-\sin^2\theta$$
$$=\cos^2\theta-(1-\cos^2\theta)$$
$$=2\cos^2\theta-1$$

Q11

$$sin^6\theta+cos^6\theta$$
$$=(sin^2\theta)^3+(cos^2\theta)^3$$
$$=(sin^2\theta+cos^2\theta)(sin^4\theta+cos^4\theta-sin^2\theta cos^2\theta)$$
$$=\left(1\right)\left((sin^2\theta+cos^2\theta)-2sin^2\theta cos^2\theta-sin^2\theta cos^2\theta\right)$$
$$=1-3sin^2\theta\cos^2\theta$$

Q12
$$\cos^2 13^\circ - \sin^2 77^\circ$$
$$= \cos^2 (90-77)^\circ - \sin^2 77^\circ$$
$$= \sin^2 77^\circ - \sin^2 77^\circ$$
$$= 0$$

LEVEL -2

Q1 base=15, $\tan\theta = \frac{8}{15}$

Prep=8

Hypotenuse= $\sqrt{15^2+8^2}$ =17

sin θ=8/17

cos θ=15/17

$$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$$

$$= \frac{(2+2\frac{8}{17})(1-\frac{8}{17})}{(1+\frac{15}{17})(2-2\frac{15}{17})}$$

=225/64

Q2 $\cot\theta = \frac{\sqrt{3}}{1}$

$$B = \sqrt{(13)^2 - (12)^2} = \sqrt{25} = 5$$

$$= \frac{\cos ec^2\theta + \cot^2\theta}{\cos ec^2\theta - \sec^2\theta} = \frac{\left(\frac{2}{1}\right)^2 + \left(\frac{\sqrt{3}}{1}\right)^2}{\left(\frac{2}{1}\right)^2 - \left(\frac{\sqrt{3}}{1}\right)^2}$$

=7/1X3/8=21/8

Q3 $\cos ec\theta = \frac{13}{12}$

H=13, P=12

$$B = \sqrt{(13)^2 - (12)^2} = \sqrt{25} = 5$$

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$$

$$\frac{2\frac{12}{13}-3\frac{5}{13}}{4\frac{12}{13}-9\frac{5}{13}} = \frac{9}{13} X \frac{13}{3} = 3$$

Q4 $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ) = 6$

$= 2(\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2 - 6(\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2)$

= $2(\left(\frac{1}{2}\right) + (3)) - 6(\left(\frac{1}{2}\right) - \left(\frac{1}{3}\right))$

=7-1

=6 =RHS

Q 5 $2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ$

Q6 $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

$= \frac{\frac{1}{2} - 1 + 2x1}{\frac{1}{\sqrt{3}} x\sqrt{3}}$

$= \frac{\frac{3}{2}}{1}$

= $\frac{3}{2}$

Q 7 $\cos 65^\circ \sin 25^\circ + \cos 25^\circ \sin 65^\circ$

$= \cos(90 - 25)^\circ \sin 25^\circ + \cos 25^\circ \sin(90^\circ - 25^\circ)$

$= \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ$

$= \sin^2 25^\circ + \cos^2 25^\circ$

=1

Q 8 $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$= \frac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2(90^\circ - 73^\circ) + \cos^2 73^\circ}$

$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$ =1

Q 9 $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$ Take LHS= $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$

$= \tan(90^\circ - 80^\circ)\tan(90^\circ - 75^\circ)\tan 75^\circ \tan 80^\circ$

$= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ$

=1

Q 10 Take LHS

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} X \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} X \frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin\theta-\cos\theta(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

=Divide each term by $\sin\theta\cos\theta$

RHS

Q11Prove that $(\cos ec\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$

Take LHS

$= (\cos ec\theta - \sin\theta)(\sec\theta - \cos\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta}\frac{\sin^2\theta}{\cos\theta}$$

$= \sin\theta\cos\theta$

Take RHS

$= \frac{1}{\tan\theta + \cot\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$

$= \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$

$= \sin\theta\cos\theta$

LHS=RHS

Level 3

Q 1 $\sqrt{3}\tan\theta = 3\sin\theta$

$= \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$= \frac{\sqrt{3}}{3} = \frac{\sin\theta\cos\theta}{\sin\theta}$

$\cos\theta = \frac{1}{\sqrt{3}}$

$\cos^2\theta = \frac{1}{3}$

$\sin^2\theta - \cos^2\theta = 1 - 2\cos^2\theta$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

Q 2 $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\cos\theta = \sqrt{2}\sin\theta - \sin\theta$

$\cos\theta = (\sin\theta)\left(\sqrt{2} - 1\right)$

$\frac{\cos\theta}{\sin\theta} = \left(\sqrt{2} - 1\right)$

$\cot\theta = \left(\sqrt{2} - 1\right)$

Q 3 $\sin(A + B) = 1$

$A + B = 90^\circ$

$\cos(A - B) = \frac{\sqrt{3}}{2}$

$A - B = 30^\circ$

$2A = 120^\circ$

$A = \frac{120^\circ}{2}$

$A = 60^\circ$

$B = 30^\circ$

Q 4 $\tan\theta + \cot\theta = 2$

Sqaring both sides

$(\tan\theta + \cot\theta)^2 = (2)^2$

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$

$\tan^2\theta + \cot^2\theta = 4 - 2$

$\tan^2\theta + \cot^2\theta = 2$

Q 5 $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

Take LHS

$= \cos 2\theta$

$= \cos 2x30^\circ$

$= \cos 60^\circ$

Take RHS

$\frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$

$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$

$=\dfrac{\frac{2}{3}}{\frac{4}{3}}$

$=\dfrac{2}{3}x\dfrac{3}{4}$

$=\dfrac{1}{2}$

Q 6 $\sin 3\theta = \cos(\theta - 6)$

$\sin 3\theta = \sin\left(90^\circ - (\theta - 6)\right)$

$3\theta = 90^\circ - \theta + 6$

$3\theta + \theta = 96$

$4\theta = 96^\circ$

$\theta = 24^\circ$

Q 7 $\dfrac{2}{3}\cos ec^2 58^\circ - \dfrac{2}{3}\cot 58^\circ \tan 32^\circ - \dfrac{5}{3}\tan 37^\circ \tan 13^\circ \tan 45^\circ \tan 77^\circ \tan 53^\circ$

$=\dfrac{2}{3}\cos ec^2 58^\circ - \dfrac{2}{3}\cot 58^\circ \tan(90^\circ - 58^\circ) - \dfrac{5}{3}\tan(90^\circ - 53^\circ)\tan(90^\circ - 77^\circ)\tan 77^\circ \tan 53^\circ x1$

$=\dfrac{2}{3}\left(\cos ec^2 58^\circ - \cot^2 58^\circ\right) - \dfrac{5}{3}\cot 53^\circ \cot 77^\circ \tan 77^\circ \tan 53^\circ x1$

$=\dfrac{2}{3} - \dfrac{5}{3}$

$=-1$

Q 8 $\tan\theta \cot(90^\circ - \theta) - \sec\theta \cos ec(90^\circ - \theta) + 3\sqrt{3}\tan 13^\circ \tan 30^\circ \tan 77^\circ$

$= \tan\theta \tan\theta - \sec\theta \sec\theta + 3\sqrt{3}\tan(90^\circ - 77^\circ)\tan 77^\circ \tan 30^\circ$

$= \tan^2\theta - \sec^2\theta + 3\sqrt{3}\cot 77^\circ \tan 77^\circ \dfrac{1}{\sqrt{3}}$

$= -1 + 3$

$=2$

Q 9 Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Take LHS

$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\left((\sin^2\theta)^3 + (\cos^2\theta)^3\right) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2((\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta) - 3((\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta) + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

=0

Q 10: $(\sin A - \cos A)(1 + \cot A + \tan A)$

Sol: $sinA + cotAsinA - cosAcotA - cosA + tanAsinA - cosAtanA$

$$= sinA - cosA + sinA * \frac{cosA}{sinA} + sinAtanA - cosAcotA - cosA * \frac{sinA}{cosA}$$

$= sinAtanA - cosAcotA$

Q11: Sol: $L.H.S = \left(sinA - \frac{1}{sinA}\right)(cosA - \frac{1}{cosA})$

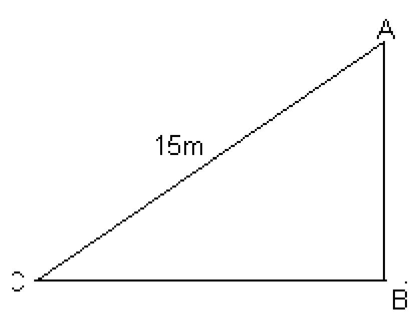
$$\left(\frac{sin^2A - 1}{sinA}\right)\left(\frac{cos^2A - 1}{cosA}\right) = \left(-\frac{cos^2A}{sinA}\right)\left(\frac{-sin^2A}{cosA}\right)$$

R.H.S $\frac{1}{tanA+cotA}$

$$=\frac{1}{\frac{sinA}{cosA}+\frac{cosA}{sinA}} = \frac{1}{\frac{sin^2A+cos^2A}{sinA.cosA}} = \frac{sinAcosA}{sin^2A+cos^2A} = \frac{sinAcosA}{1} = sinAcosA = L.H.S$$

SOME APPLICATION OF TRIGONOMETRY

Q1



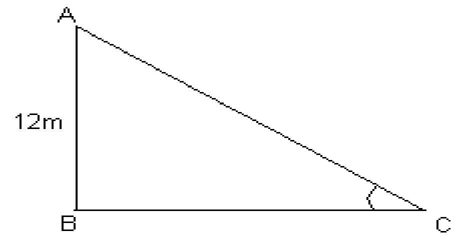
Let x be the height of the wall

then x/15=sin60= $\sqrt{3}$/2

$x = 15\frac{\sqrt{3}}{2} = 7.5\sqrt{3}$ m

Height of the wall is 7.5$\sqrt{3}$ m

Q2

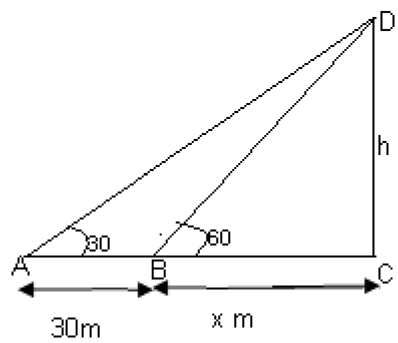


Let θ be the angle of elevation

Then $\tan\theta = 12/(4\sqrt{3}) = 3/\sqrt{3} = \sqrt{3}$

$\theta = 60$

Q3



Let height of the tower be h.Č

Let BC=x

Then tan 60=h/x

$h = x\sqrt{3}$ ……………………………………………….(1)

$x = h/\sqrt{3}$ ……………………………………………….(2)

tan30=h/(x+30)

$$\frac{1}{\sqrt{3}} = \frac{h}{x+30}$$

$$h = \frac{x+30}{\sqrt{3}}$$

From(1)and(2)

$$h = \frac{\frac{h}{\sqrt{3}} + 30}{\sqrt{3}}$$

$$= \frac{h + 30\sqrt{3}}{3}$$

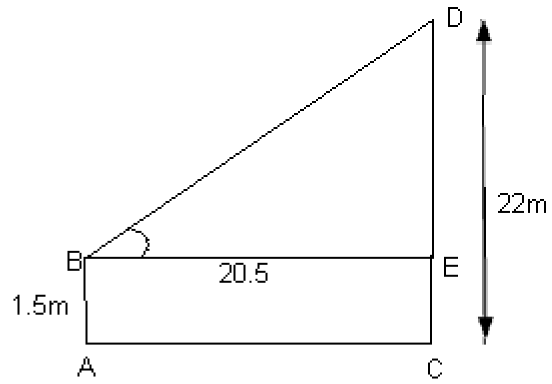
$3h = h + 30\sqrt{3}$

$3h - h = 30\sqrt{3}$

$h = 15\sqrt{3}$ m

LEVEL 2

Q1



Let AB=1.5m

DC=22m

CE=1.5m

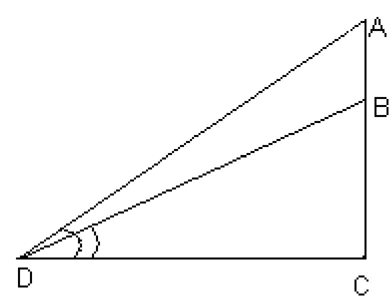
DE=22-1.5=20.5m

In $\sqcup$ BCD Let $\angle$DCB=θ

tanθ=DE/BC=1

θ=45

Q2



Let Ac be the height of the aeroplane. Then AC=5000m

Let B be the other aeroplane over which plane A passes vertically.

Then tan60=AC/DC=5000/ $\sqrt{3}$

$\sqrt{3}$=5000/DC or DC=5000/$\sqrt{3}$

tan45=BC/DC

1=BC/DC

BC=DC

Then distance between aeroplane=AC-BC

=5000-5000/$\sqrt{3}$

=5000(1-1/$\sqrt{3}$)

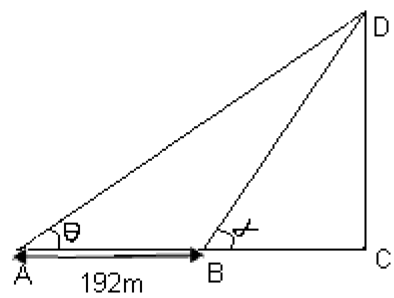
=5000x$\frac{5}{12}=\frac{3}{4}$

=5000(1.27/3)

=2116.5m

Q3 solution Let A be the point and angle of elevation is θ

Such that $\frac{5}{12}$



After walking 192m towards tower CD, $\angle DBC =$ α SUCH that tan α=3/4

Then $\tan\theta = \frac{DC}{AC} = \frac{DC}{192+BC}$

$\frac{5}{12} = \frac{DC}{192+BC}$

960+5BC=12DC

960=12DC-5BC (i)

tan α= $\frac{DC}{BC}$

$\frac{3}{4}$ = $\frac{DC}{BC}$

3BC=4DC

4DC-3BC=0

12DC-9B C=0 (ii)

Solving (i) and (ii)

12DC-5BC =960

\- <u>12DC- $_{+}$9B C=0</u>

4BC=960

BC=240

Put in eq(i)

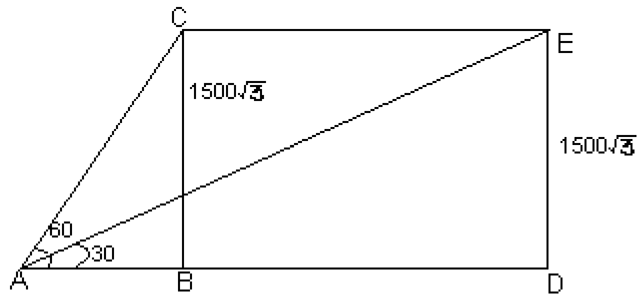
12DC- 5(240) =960

12DC=2160

DC=180 m

Level 3

Q1



In ΔADE

$$\frac{DE}{AD} = \tan 30^\circ$$

$$\frac{1500\sqrt{3}}{AD} = \frac{1}{\sqrt{3}}$$

$$AD = 1500\sqrt{3} x \sqrt{3}$$

AD=4500m

InΔCBA

$$\frac{BC}{AB} = \tan 60^\circ$$

$$\frac{1500\sqrt{3}}{AB} = \sqrt{3}$$

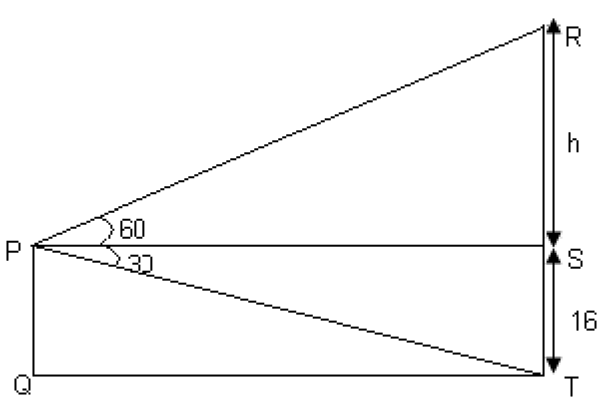
AB=1500

Distance EC travelled in 15sec=4500-1500=3000m

Speed of aero plane=3000/15=200m/sec

Q2 Let the height of the deck=PQ=16m

Let the height of cliff =RS+ST=h+16



In triangle PSR

$$\frac{RS}{PS} = \tan 60^\circ =$$

$$\frac{RS}{PS} = \sqrt{3}$$

$$\frac{h}{PS} = \sqrt{3}$$

$$\frac{h}{\sqrt{3}} = PS$$

In triangle PST

$$\frac{ST}{PS} = \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

PS=16 $\sqrt{3}$

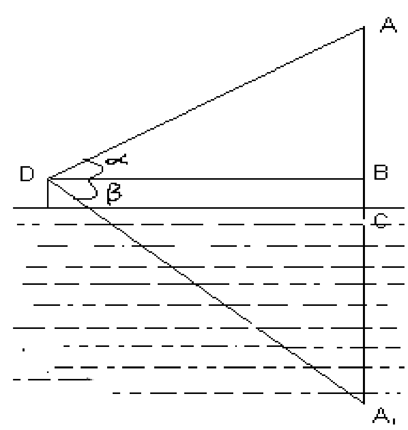
$\frac{h}{\sqrt{3}}$ =16 $\sqrt{3}$

h=48m

The total height of cliff=16+48=64

QT=PS=16 $\sqrt{3}$

Q3 Let D be the point of observation. A be the position of cloud



A_1 be the image of cloud A

From□ ABD

$\frac{x-h}{BD} = \tan\alpha$ ....(i)

$$BD = \frac{x-h}{\tan\alpha}$$

In triangle □A_1BD

$$\frac{x+h}{BD} = \tan\beta$$

$$BD = \frac{x+h}{\tan\beta}$$

$\tan\beta = \dfrac{x+h}{BD}$ ……………………………………………….(ii)

Subtracting (i) from (ii)

$\tan\beta - \tan\alpha = \dfrac{1}{BD}\left[x+h-x+h\right]$

$= \dfrac{2h}{BD}$

$BD = \dfrac{2h}{\tan\beta - \tan\alpha}$ ………………………………..(iii)

Also in ΔABD

$\dfrac{AD}{BD} = \sec\alpha$

$BD = \dfrac{AD}{\sec\alpha}$ ……………………………………………….(iv)

Equating (iii) and (iv)

$\dfrac{AD}{\sec\alpha} = \dfrac{2h}{\tan\beta - \tan\alpha}$

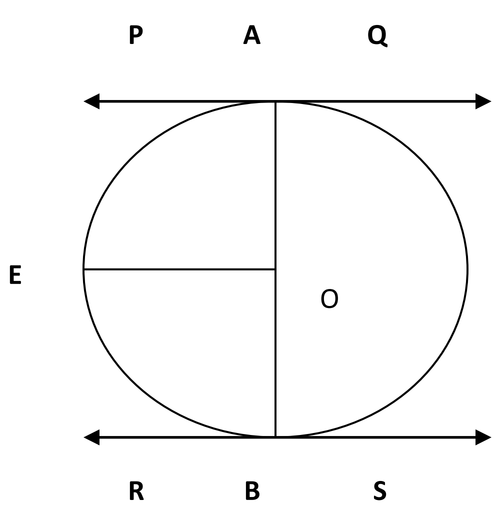
$AD = \dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

Hence distance of cloud A from point of observation is $\dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

CIRCLES

LEVEL I

1.



Angle PAO = 90°

Since, PA is parallel to EO

Hence,angle PAO + angle EOA = 180°

Or angle EOA = 90°

Similarly,angle EOB= 90°

So, angle EOA + angle EOB = 180°

That's why AOB is a straight line

Hence AOB is a diameter.

2. Angle OAT= 90°

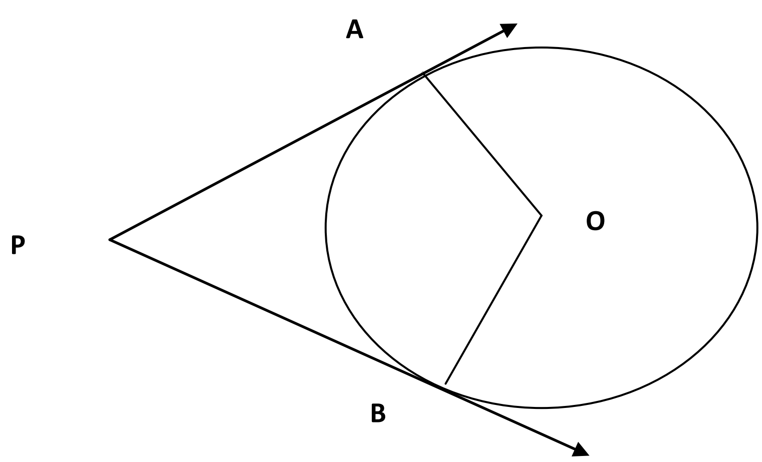
In right triangle AOT by Pythagoras theorem,

AT = $\sqrt{(OT^2 - AT^2)}$

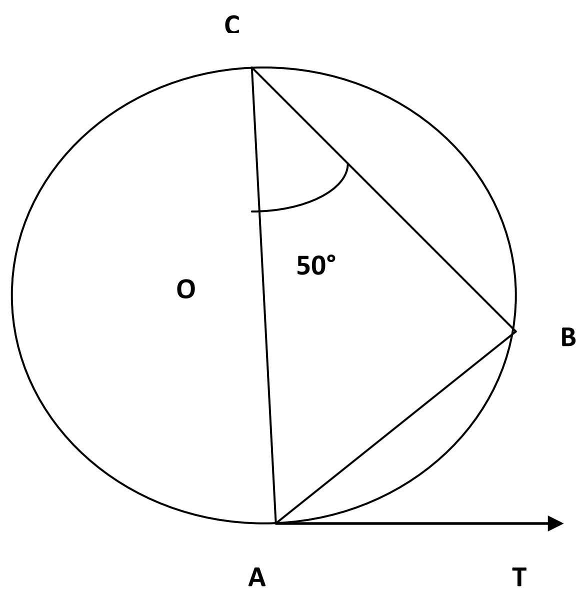
Or AT = 12 cm.

3. Angle OAP = 90° and angle OBP= 90°

Hence angle APB = 50° by angle sum property of a quadrilateral



4.



Angle ABC = 90°   [angle in a semi circle is right angle]

By angle sum property in triangle ABC, angle BAC = 40°

Since the radius of a circle is perpendicular to the tangent at the point of contact

Hence angle OAT = 90°

Or angle BAC + angle BAT = 90°

Or angle BAT = 50°

5. XP = XQ ------ (i)
   AP = AR ------ (ii)

   BQ = BR ------ (iii)

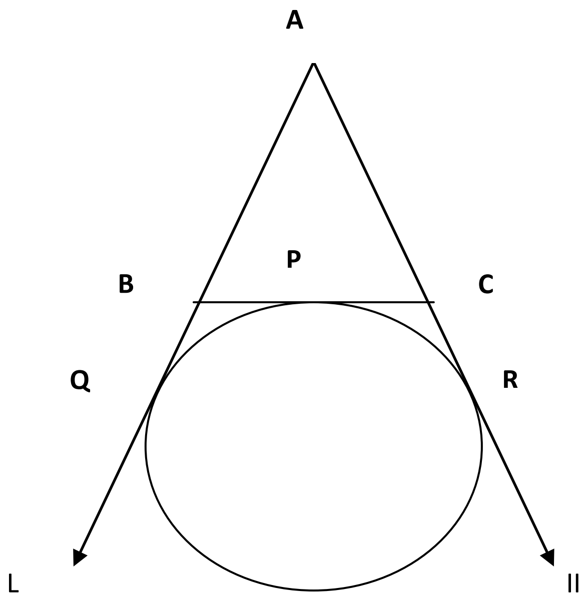
   Now from (i), XP = XQ

   Or XA + AP = XB + BQ

   Or XA + AR = XB + BR [from (i) and (ii)]

LEVEL II

1.



Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence AQ = AR , BP = BR and CP = CR

Now perimeter of triangle ABC = AB + BC+ AC

= AB + BP + CP + AC

= AB + BQ + CR + AC

= AQ + AR

= AQ + AQ

= 2AQ

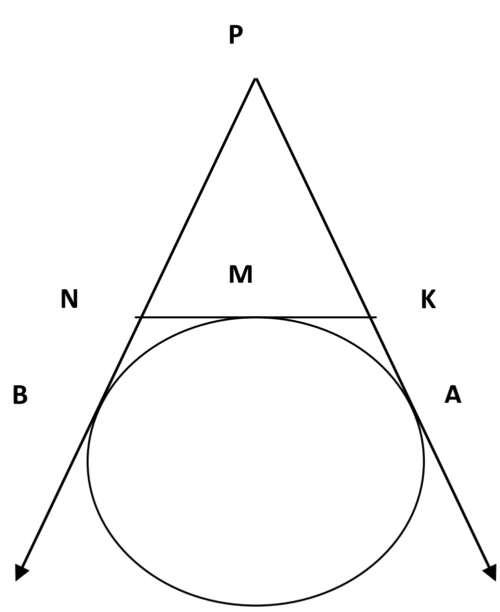
Hence AQ = ½ Per (triangle ABC)

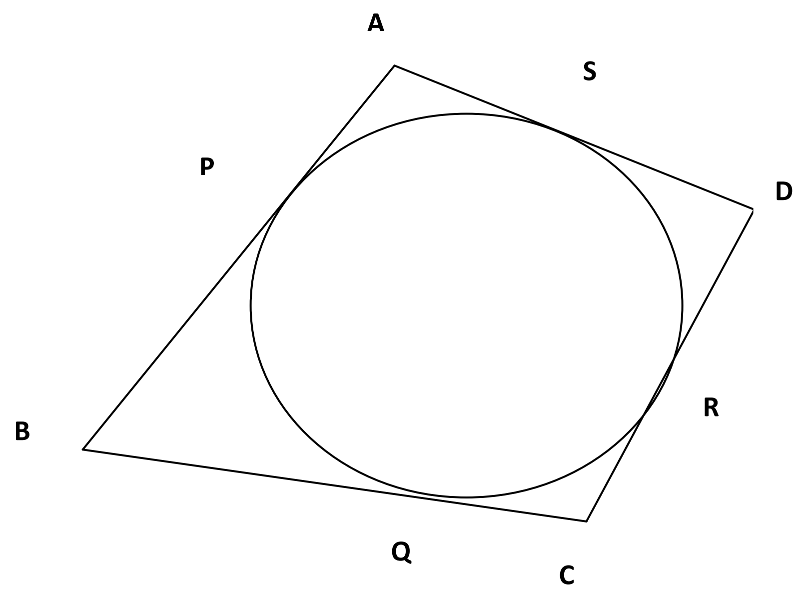
2. Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence KM = KA and NM = NB

Now KN = KM + MN

= KA + NB





3.

Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence AP = AS ------I

BP = BP ------ II

CP = CQ ------ III

DR = DS ------ IV

Adding I, II, III and IV,

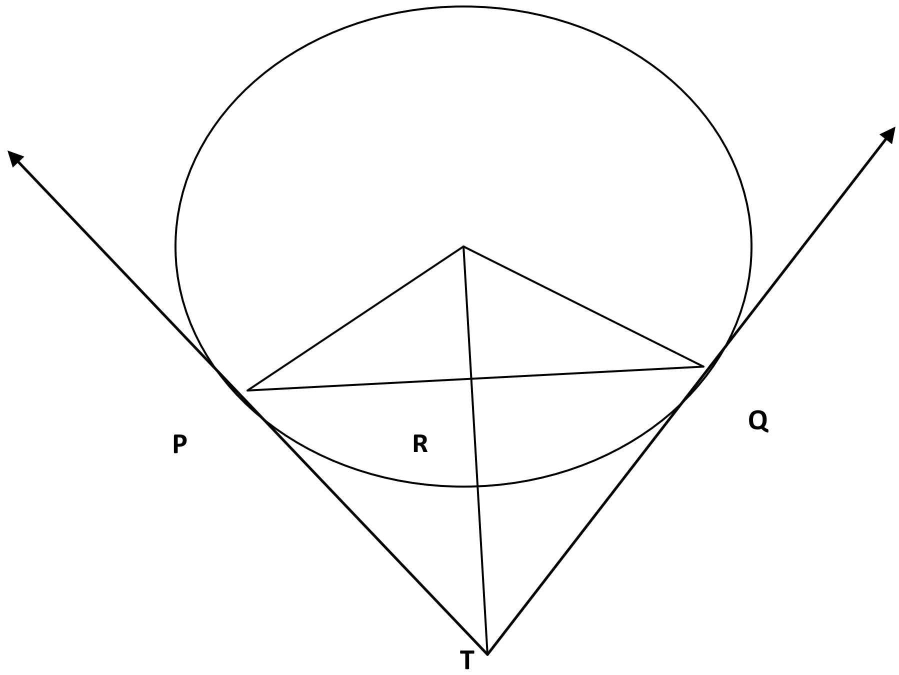
AP + BP + CR + DR = AS + BQ + CQ + DS

Or AB + CD = AD + BC

Or 6 + 4 = AD + 7

Or AD = 3 cm

Q4.



Here PR = RQ = 4 cm

In right triangle POR,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2}$$

$$= 3 \text{ cm}$$

Now angle OPT = 90°

Or angle OPR + angleTPR = 90°

Also angleTPR + anglePTR = 90°

Therefore angleOPR = anglePTR

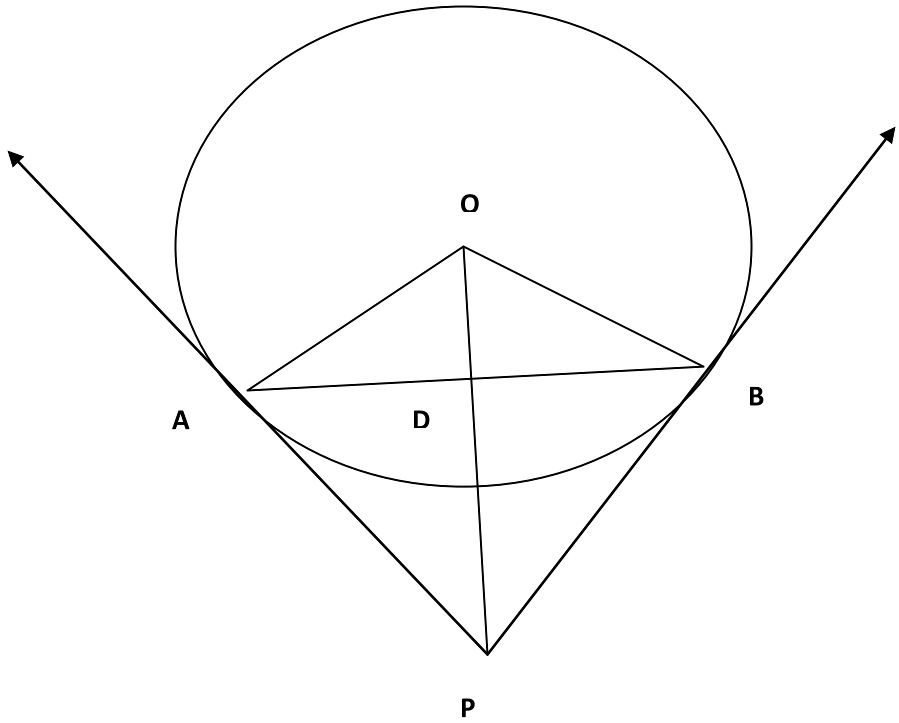
Hence right triangle TRP is similar to right triangle PRO

Or TP/PO = RP/RO

Or TP/5 = 4/3

Or TP = 20/3 cm

5.



It can be proved that triangle PAO is congruent to triangle PBO

Hence anglePAO = anglePBO

Now trianglePAD is congruent to triangle PBD

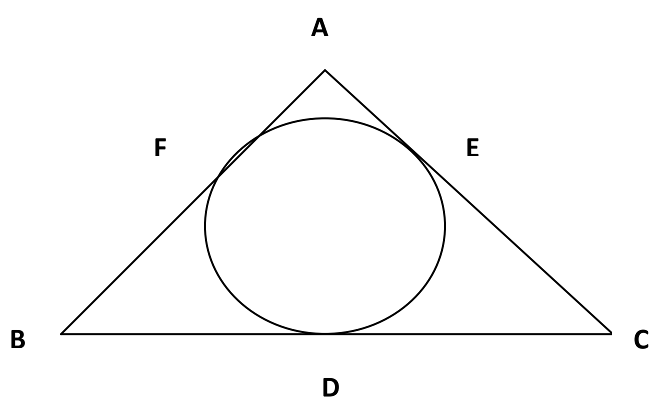
Hence AD = BD -------(I)

And angle ADP = angleBDP

But angle ADP + angleBDP ------- (II)

From (I) and (II), PO is perpendicular bisector of AB

6.



Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence AF = AE ------ I

BD = BF ------II

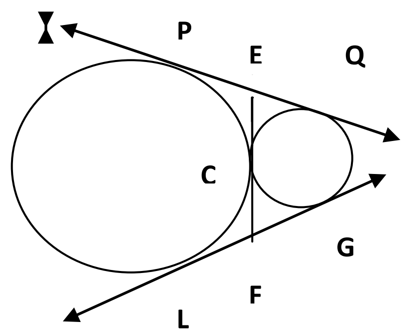
CE = CD ------III

Adding I, II and III,

AF + BD + CE = AE + BF + CD

LEVEL III

1. Since the lengths of the tangent segments drawn from an external point to a circle are equal. Hence EP = EC and EQ = EC
   Therefore EP = EQ



Similarly FL = FG

Therefore common tangent EF bisects other common tangents PQ and LG.

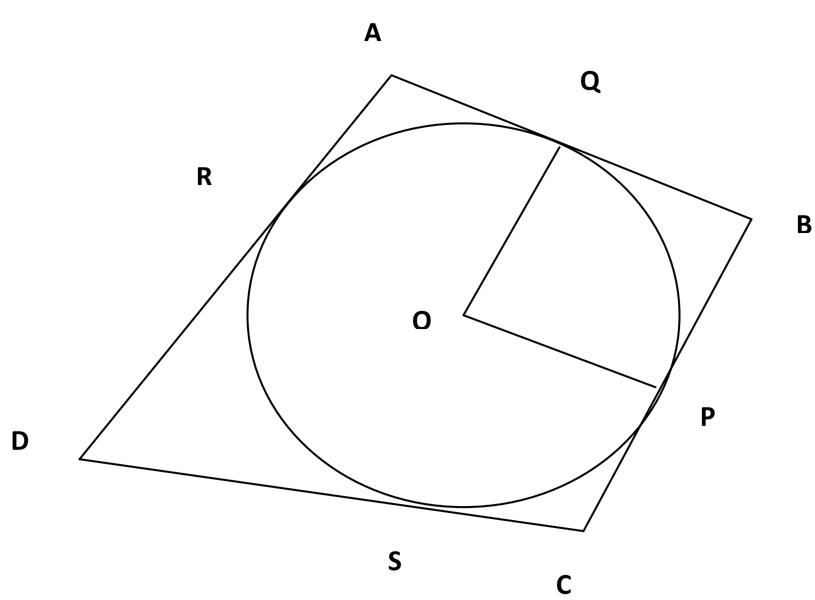
2. It is clear that quadrilateral OBPQ is a square.
   Hence OQ = OP = PB = BQ = r

   Since the lengths of the tangent segments drawn from an external point to a circle are equal. Hence DS = DR = 5 cm

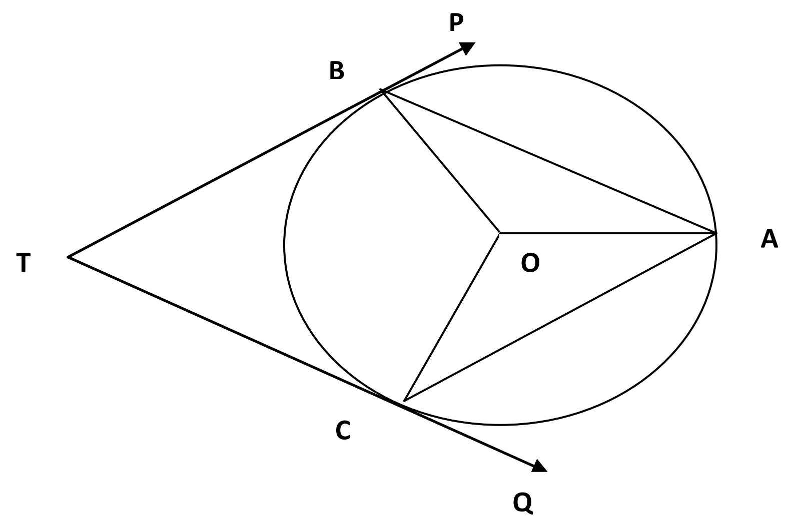
   AR = AQ = 23-5 = 18cm

   BQ = BP = 29 – 18 = 11cm

   Therefore r = 11 cm.



3.



Since the radius of a circle is perpendicular to the tangent at the point of contact

Hence angle OBA + angleABP = 90°

Or angle OBA + 60° = 90°

Or angle OBA = 30°

Similarly angle OCA = 20°

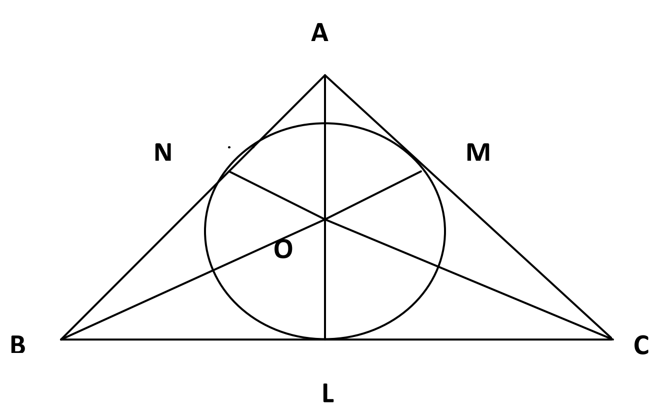
Since triangles AOB and AOC are isosceles triangles

Hence angle OAB = 30° and angle OAC = 20°

Therefore angle BAC = 30° + 20°

= 50°

4.



Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence AM = AN =6 cm, CM = CL = 8 cm and BL = BN = x cm

By Heron's formula area of triangle ABC = $\sqrt{48x\,(x + 4\ 0)}$

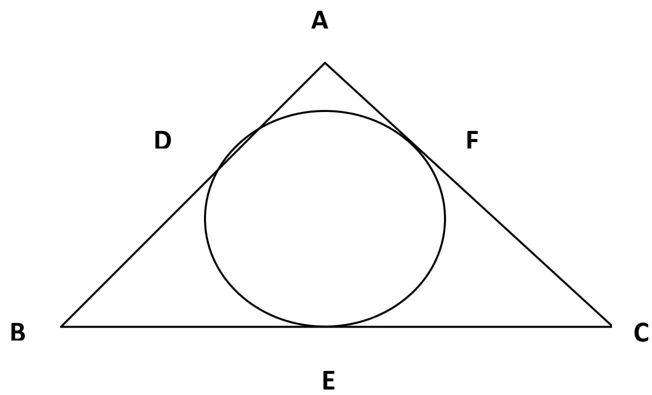
Also area of triangle ABC = area of triangle BOC + area of triangleCOA + area of triangleAOB

Or $\sqrt{48x\,(x + 4\ 0)} = \frac{1}{2}(8 + x) \times 4 + \frac{1}{2}(8 + 6) \times 4 + \frac{1}{2}(6 + x) \times 4$

Or $x = 7$ cm

Hence BC = 15 cm and AB= 13 cm

5.



Since the lengths of the tangent segments drawn from an external point to a circle are equal.

Hence AD = AF ------ I

BE = BD ------II

CF = CE ------III

Adding I, II and III,

AD + BE + CF = AF + BD + CE ------- IV

Now, perimeter of triangleABC = AB + BC + CA

= AD + BD + BE + CE + CF + AF

= (AF + BD + CE) + (AD + BE + CE)

= 2(AF + BD + CE)

Therefore, (AF + BD + CE) = ½ perimeter of triangleABC

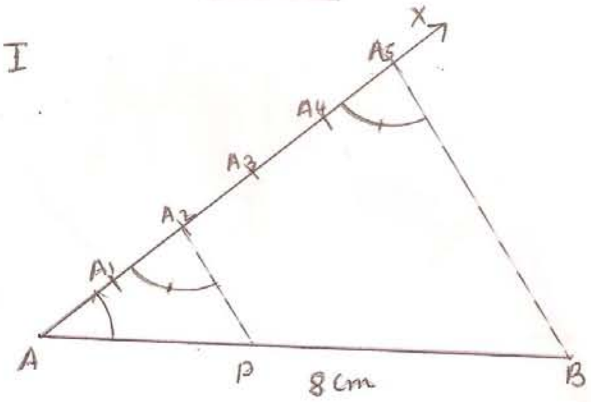
CONSTRUCTIONS

CONSTRUCTIONS

SOLUTIONS

LEVEL I

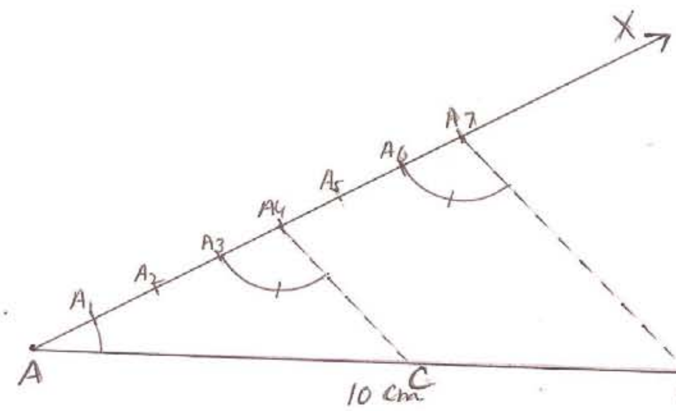
1.



∠BOX is any acute angle

AP : PB = 2 : 3.

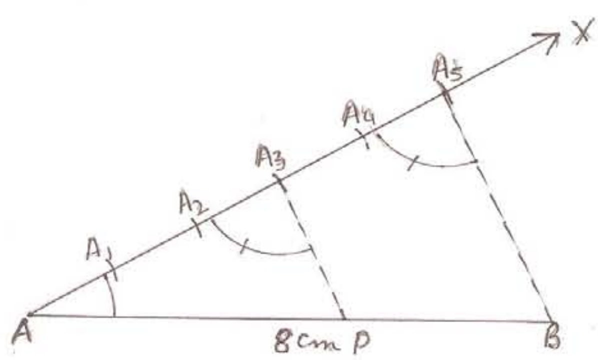
2



∠BAX is any acute angle

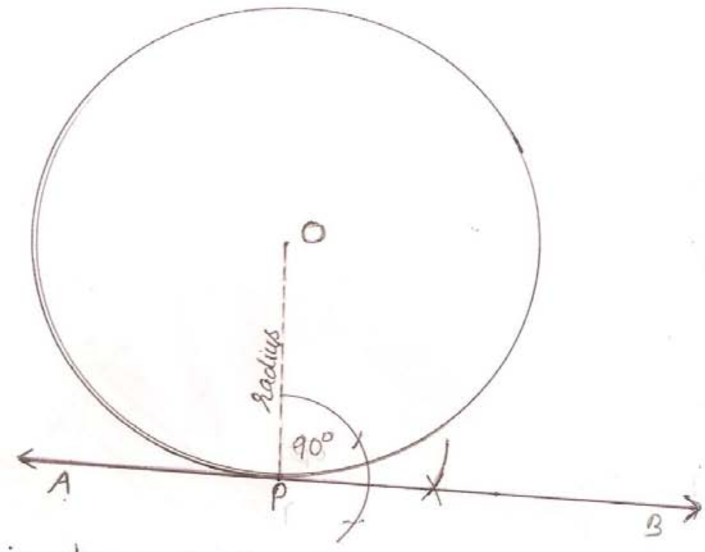
AC : CB = 4 : 3

3



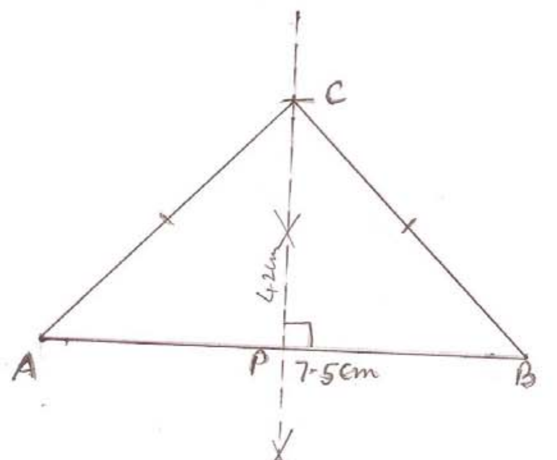
$AP = \frac{3}{5} AB$

4



AB is tangent to the circle at point P.
to the circle.

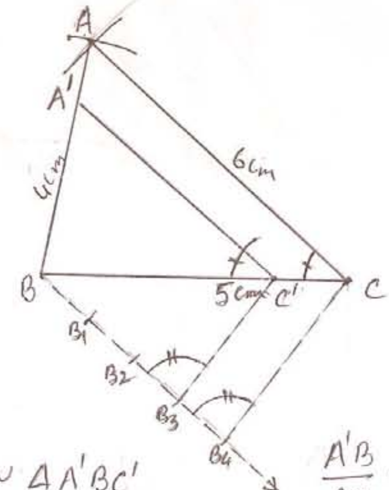
5



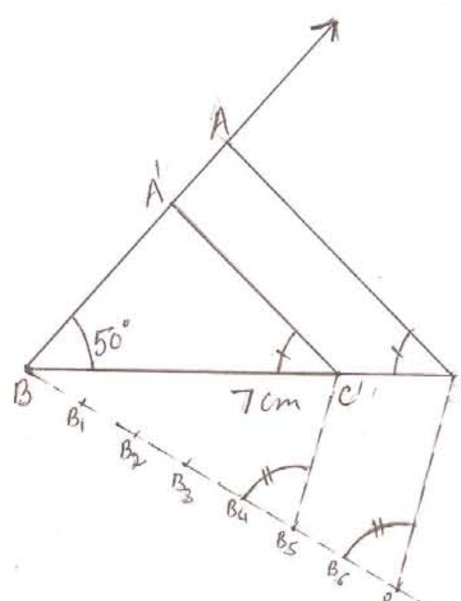
Δ ABC is an isosceles triangle where ∠BPC = 90°, CP = 4.2cm
CP is the altitude

LEVEL : II

1

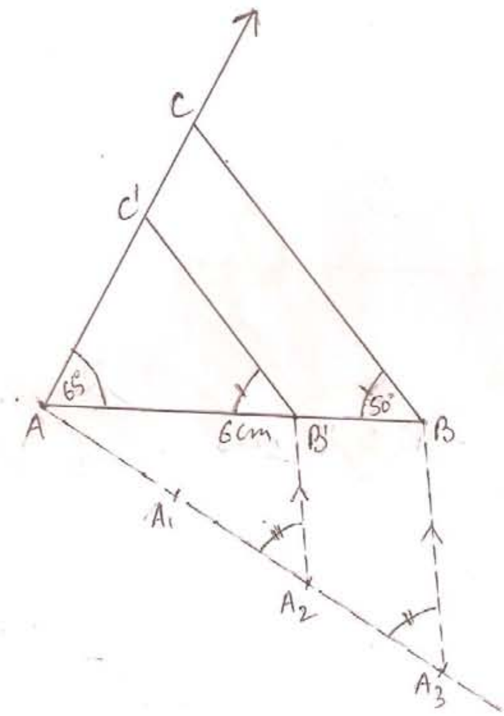


$\Delta ABC \sim \Delta A'BC'$ $\quad \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AB} = \frac{3}{4}$



$\Delta A'BC' \sim \Delta ABC$, Where $\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{5}{7}$

3



$\angle B = 50°$

$\angle C = 65°$

$\angle A = 180 - (\angle B + \angle C)$

$\angle A = 180 - (50° + 65°)$

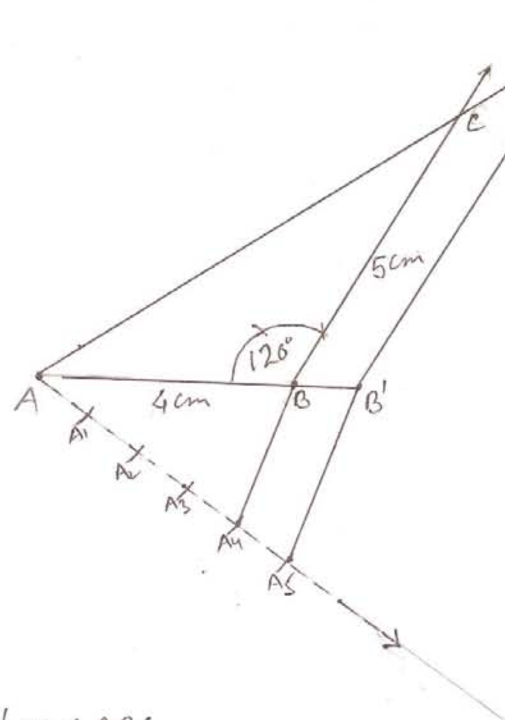
$= 180 - 115°$

$\angle A = 65°$

$\Delta AC'B' \sim \Delta ACB$

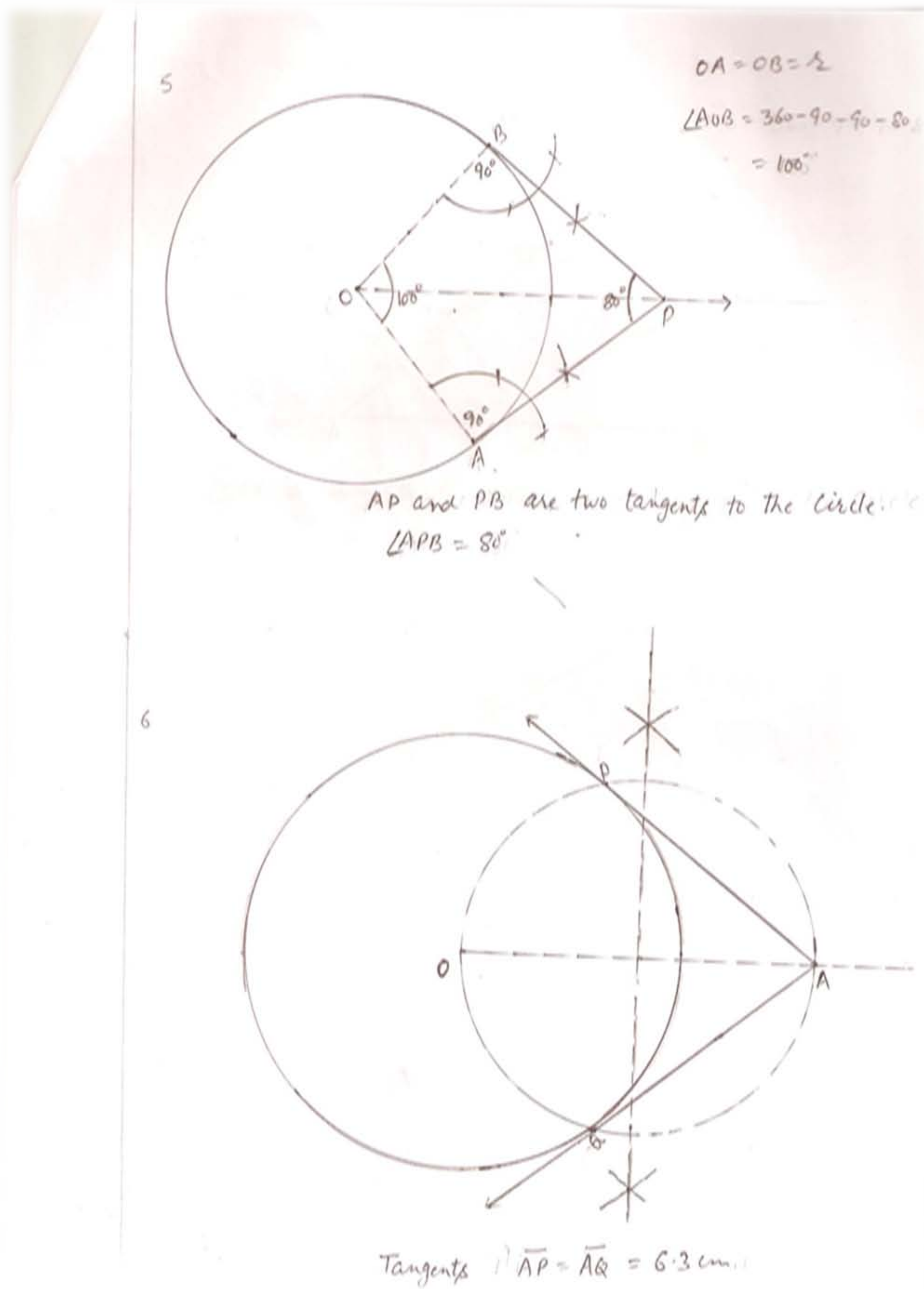
$$\frac{AC'}{AC} = \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{2}{3}$$

4



$\Delta AB'C' \sim \Delta ABC$

$$\frac{AC'}{AC} = \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{5}{4}$$



AREA RELATED TO CIRCLE

LEVEL I

Q1. r =21cm

θ =150^0

Length of an arc = θ/360 x2 π r

$=150/360 \times 2 \times 22/7 \times 2$

$=55cm$

Q2. Circumference of circle = 22cm

$2r\pi$ = 22cm

r = 22x7/2x22

=3.5cm

Area of circle $=\pi r^2$

$=22/7x(3.5)^2$

$=38.5\ cm^2$

Q3. Area of circle $=301.84cm^2$

πr^2 =301.84

R^2 =301.84x7/22

R = 9.8cm

Circumference of circle = $2\pi r$

=2x22/7x9.8

=61.6 cm

Q4. Distance covered in 5000 revolutions =11km

=11000m

Distance covered in 1 revolution =11/5m

Circumference of circle = 11/5m

$2\pi r$ = 11/5

R =11/5x7/2x1/22

R 7/20m

So, d =2x7/20

=7/10m

=70cm

Q5. Length of an arc $=20\pi$ cm

θ $=144^0$

$2\pi r\,\theta/360 = 144^0$

$2\pi r\,144/360 = 20\pi$

R =25cm

LEVEL II

Q1. r = 5cm

Area of sector = 5 π cm^2

$\pi r^2 \theta/360$ = 5 π

$5^2 \theta/360$ = 5

θ = 72^0

Q2. r = 5.6m

Perimeter = 27.2m

R + r+ length = 27.2m

2 x5.6 + 2 π r θ/360 = 27.2

2x π 5.6x θ/360 = 16

θ = 1800/11

Area of sector = θ/360x π r^2

= 1800/11x1/360x22/7x$(5.6)^2$

= 44.8 m^2

Q3. Area of minor sector = ar(sectorAOB) –ar(ΔABC)

In Δ AOB

AO=OB=14cm, $\angle$ ABO= $\angle$ BAO=60^0

Δ AOB is an equilateral triangle.

ar(ΔAOB) = $\frac{\sqrt{3}}{4}(\text{side})^2$ = $\frac{\sqrt{3}}{4}(14)^2$ =49 $\sqrt{3}$ sq.cm.

Ar(sectorAOB) = $\frac{\theta}{360} \times \pi r^2$

= $\frac{60}{360} \times \frac{22}{7}(14)^2$

= 102.67 sq.cm.

Ar(minor segment) = 102.67 - 49 $\sqrt{3}$

= 17.90 sq.cm

Ar(major segment) = ar(Circle) – ar(minor segment)

= $\pi(14)^2$ – 17.90

=616 – 17.90

= 598.10 sq.cm.

Q4 ar(minor segment of circle)= ar(sector AOB) – ar(Δ AOB) ar(sector AOB) = $\frac{90}{360} \times \pi\,(14)^2$

$= \frac{22}{7} \times \frac{14 \times 14}{4}$

= 154 sq.cm

Ar(ΔAOB) = $\frac{1}{2} \times 14 \times 14$ = 98 sq. cm.

Ar(minor segment) = ar(circle) – ar(minor segment)

$= \pi\,(14)^2 - 56$ = 560sq.cm

Q5. , PQ=12cm, θ=120^0 and OM$\perp$PQ.

In ΔPOQ

OP=OQ

$\angle$POQ=120^0

$\angle$OPQ=$\angle$OQP=30^0

So, PM=MQ= 6cm

In right angled ΔOPM,

$\frac{PM}{OP}$= Cos 30^0

$\frac{6}{OP} = \frac{\sqrt{3}}{2}$

OP=4$\sqrt{3}$ CM

$\frac{PM}{OP}$=Sin30^0

$\frac{OM}{OP} = \frac{1}{2}$

OM = $2\sqrt{3}$

Radius of circle , r= OP= 4$\sqrt{3}$

Ar(sector) = $\frac{\theta}{360} \times \pi$ r^2

$= \frac{120}{360} \times \pi\,(4\sqrt{3})^2$

=16π sq.cm

Ar(ΔPOQ) = $\frac{1}{2} \times$ AB $\times$ OM =12$\sqrt{3}$ sq.cm

Ar(minor segment)= 16π - 12$\sqrt{3}$

= 4(4π - 3$\sqrt{3}$) sq.cm.

Q6. Radius of O = 4cm

AB =BC =CA

So, ΔABC is an equilateral Δ.

$\angle$ AOC = 120^0

$\angle$ AOC = 120^0

θ = 120^0

Ar (sectorAOC) = $\frac{120}{360} \times \pi\, 4^2$

= $\pi/3 \times 16$

= $\frac{16}{3} \times \pi$

Ar (ΔAOC) = $r^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$

= $4^2 \sin\frac{120}{2} \cos\frac{120}{2}$

= 16 sin60 cos60

= $16 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$

= $4\sqrt{3}$ cm^2

Area of shaded region = $\frac{16}{3}\pi$ --$4\sqrt{3}$

= $4(\frac{4}{3}$ --$\sqrt{3}$) cm^2

LEVEL III

Q1: In rtΔ ABC , $\angle C = 90^0$

Base =AC =12cm, $\perp$ $\boldsymbol{Bc = 16cm}$

By Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

$= 12^2 + 16^2 \;\; = 144 + 256 = 4cm$

$AB = \sqrt{400} = 20cm$

Radius OA = 10cm

Area of semi –circle = $\frac{\pi r^2}{2} = \frac{3.14}{2} * 10 * 10 = 157cm^2$

Area of rt $\Delta\, ABC = \frac{1}{2} * base * height = \frac{1}{2} * 12 * 6 = 96\; cm^2$

Area of shaded portion =157-96 =61cm^2

Length of semi –circle arc ACB =$\frac{1}{2} * 2\pi r = 3.14 * 10 = 31.4\; cm$

Perimeter of shaded portion =31.4+12+16=59.4cm

Q2. Diameter of circle A,B,C = 3CM, R = 1.5 cm

Radius of semicircle E = 4.5 cm

Radius of circle D = 2.25 cm

Ar (shaded region) = ar (semicircle E) – ar(semicircleA) – ar (semicircleC)

--ar(circle D) – ar(semicircleB)

$$= \frac{1}{2}\pi(4.5)^2 - \frac{1}{2}\pi(1.5)^2 - \frac{1}{2}\pi(1.5)^2 - \pi(2.25)^2 + \frac{1}{2}\pi(1.5)^2$$

$= 12.37\ cm^2$

Cost of painting shaded region = $\frac{25}{100} \times 12.37$

= Rs.3.092

= Rs. 3 (appro)

Q3. AB =14cm

BC = 7 cm

Ar(shaded region) = ar(rectangleABCD) –ar(semicircle I) + ar(semicircle II)+ Ar(semicircleIII)

$$= 14 \times 7 - \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi(3.5)^2 + \frac{1}{2}(3.5)^2$$

$$= 98 - \frac{1}{2}\pi(49) + \pi(3.5)^2$$

$$= 98 - \frac{1}{2} \times \frac{22}{7} \times 49 + \frac{22}{7} \times (\frac{35}{10})^2$$

= 98 – 77 + 38.5

= 59.5 sqcm

SURFACE AREA AND VOLUMES

LEVEL I

1. Volume of the container= $\frac{1}{3}\pi h\ (R^2 + r^2 + Rr)$

$= \frac{1}{3}\pi \times 16\ [20\times20 + 8\times8 + 20\times8]$

= 10459.43 cu. cm

= 10.45 litres

Now $l = \sqrt{h^2 + (R-r)^2}$

= 20cm

Surface area of the container= curved surface area of container + area of the base (lower end)

$= \pi l(R + r) + \pi r^2$

$= 1961.14cm^2$

2. Volume of right circular cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 5 \times 5 \times 21$

$= 1650 \text{ cm}^3$

Now curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 5 \times 21$

$= 660 \text{ cm}^2$

3. Required ratio = $\frac{volume of 1st cone}{volume of 2nd cone} = \frac{\frac{1}{3}}{\frac{1}{3}} \frac{\prod r^2 h}{\prod R^2 h}$

$= \frac{(3x)^2}{(5x)^2} \quad = \quad 9:25$

4. Circumference of base of cone=3m

Or $2\pi r = 3$

Or $r = \frac{3}{2\prod}$

Now, volume of cone= $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{3}{2\prod}\right)^2 \times h$

$= 3.818 \text{ m}^3$

5. Total canvas used = Curved Surface Area of cylinder + Curved Surface Area of cone

$= 2\pi rh + \pi rl$

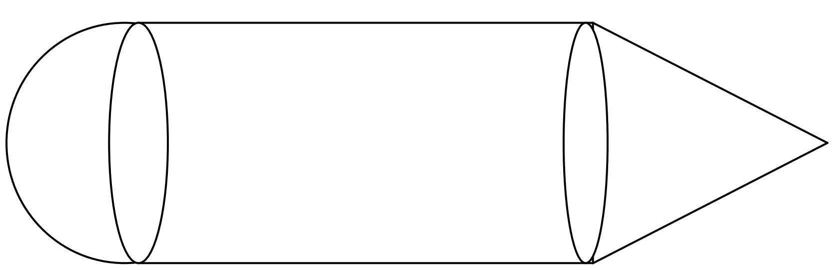
$= \pi r(2h + l)$

$= \frac{22}{7} \times 52.5 \times (6 + 53)$

$= 9735 \text{ m}^2$

LEVEL II

1. Diameter = 7 cm



r= 3.5 cm

Height of the cylindrical part, h = 6.5 cm

Since total height of the solid = 12.8 cm

Therefore height of the cone, H = 12.8 – (3.5 + 6.5)

= 2.8 cm

Now the slant height of the cone, l = $\sqrt{h^2 + (r)^2}$ = 4.47 cm

Total surface area of solid = Curved Surface Area of hemisphere + Curved Surface Area of cylinder + Curved Surface Area of cone

$= 2\pi r^2 + 2\pi rh + \pi rl$

$= \pi r(2r + 2h + l)$

$= \frac{22}{7} \times 3.5 \times (2 \times 3.5 + 2 \times 6.5 + 4.5)$

$= 269.5\ cm^2$

2. radius of the cylinder, R = 6cm

   Height of the cylinder, H = 15cm

   Let the radius of the hemisphere be r cm

   Then the height of the hemisphere (h) will be 4r cm

   Volume of ice-cream in the cylinder = 10×volume of ice-cream in one cone

or $\pi R^2 H = 10\times(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3)$

or $\pi\times6\times6\times15 = 10\times(\frac{1}{3}\pi r^2\times4r + \frac{2}{3}\pi r^3)$

or $r = 3$ cm

3. Height of cone, h =2 cm

   Diameter of the base of cone = 4cm

   Radius of the base, r = 2 cm

   Therefore the whole height of the cylinder, H = 2+2 = 4 cm

   Volume of the toy = volume of hemisphere +volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= 8\pi\ cm^3$

   Required volume of the space= volume of the cylinder – volume of the toy

$= \pi r^2 H - 8\pi$

$= 16\pi - 8\pi$

$= 8\pi\ cm^3$

4. width of the canal = 6 m

   Depth of the canal = 1.5 m

Now the length of water column per hour = 10km

The length of water column in half an hour = 5 km=5000 m

Volume of water flown in half an hour = 1.5×6 ×5000

= 45000 m^3

Standing water needed = 8 cm

= 0.08 m

Therefore area irrigated in half an hour = $\frac{volume}{height}$

= $\frac{45000}{0.08}$

= 562500 m^2

= 56.25 hectares

5. Total surface area of the article = Curved Surface Area of cylinder + 2× Curved Surface Area of a hemisphere

$= 2\pi rh + 2\times 2\pi r^2$

$= 2\pi r\,(h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5)$

= 374 cm^2

LEVEL III

1. Inner diameter of the glass = 5 cm

   Height of the glass, h= 10cm

   Therefore apparent capacity of the glass = $\pi r^2 h$

   $= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

   = 196.25 cm^3

   Volume of hemispherical part = $\frac{2}{3}\pi r^3$

   $= \frac{2}{3} \times 3.14 \times (2.5)^3$

   = 32.71 cm^3

   Therefore actual capacity of the glass = apparent capacity of glass – volume of hemispherical part

   =163.54 cm^3

2. For cylinder

Diameter = 10 cm

Radius, R = 5 cm

Height, H = 10.5 cm

For cone

Diameter = 7 cm

Radius, r = 3.5 cm

Height, h = 6 cm

i) water displaced out of the cylindrical vessel= volume of cone

$$= \frac{1}{3}\pi r^2 h$$

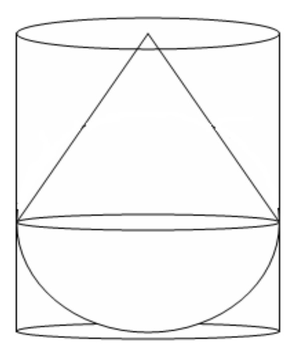
$$= 77 \text{ cm}^3$$

ii) Water left in cylindrical vessel = volume of cylinder – volume of cone

$$= \pi R^2 H - \frac{1}{3}\pi r^2 h$$

$$= 748 \text{ cm}^3$$

3. we have



Radius of hemispherical tank, r = 1.5 m

Figure 1

Volume of tank= $\frac{2}{3}\pi r^3$

$$== \frac{2}{3} \times \frac{22}{7} \times (1.5)^3$$

$$= \frac{99}{14} \text{ m}^3$$

Volume of water to be emptied= $\frac{1}{2} \times \frac{99}{14}$

$$= \frac{99}{28} \times 1000 \text{ litres}$$

$$= \frac{99000}{28} \text{ litres}$$

Since $\frac{25}{7}$ litres of water is emptied in one second. Therefore total time taken to empty half the tank i.e $\frac{99000}{28}$ litres of water

$= \frac{99000}{28} \div \frac{25}{7}$ seconds

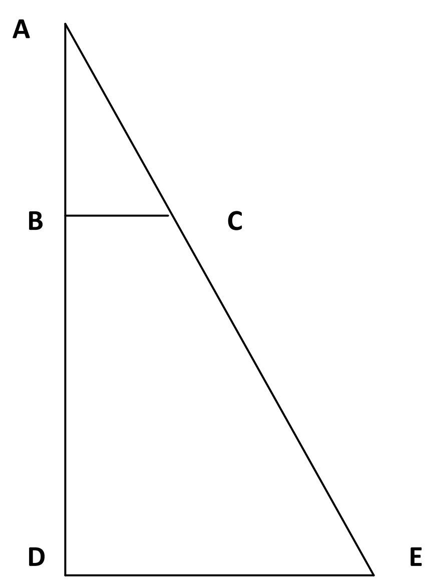
=16.5 minutes

4. AB=h

AD=30cm

BC= r cm

DE=R cm



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Triangle ABC is similar to triangle ADE

Therefore, $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{30}{h} = \frac{R}{r}$

It is given that

Volume of smaller cone = $\frac{1}{27}$(volume of bigger cone)

Or $\frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 H$

Or $\left(\frac{r}{R}\right)^2 \times h = \frac{10}{9}$

Or $\left(\frac{h}{30}\right)^2 \times h = \frac{10}{9}$

Or h = 10 cm

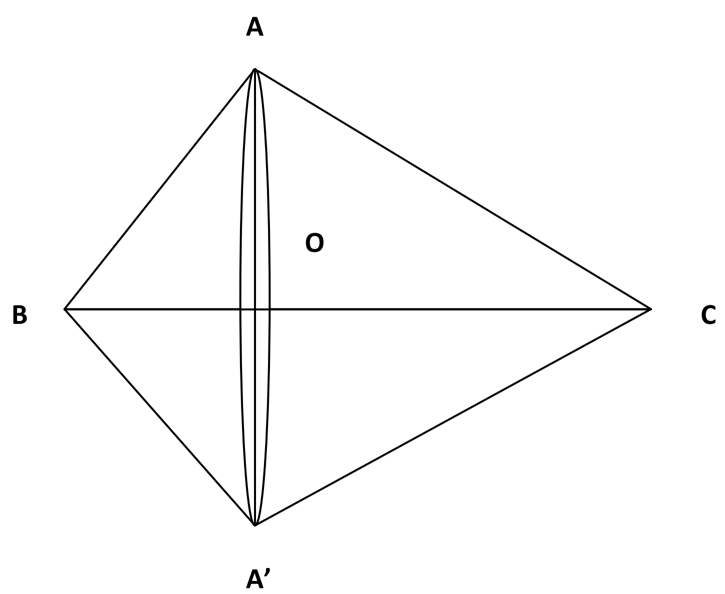
Required height = H – h

= 30 – 10

= 20 cm

5. AB= 15 cm

AC =20 cm



Radius of each cone of double cone be r cm

In right triangle BAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

Or $BC^2 = 15^2 + 20^2$

Or BC = 25 cm

Let OB=x , OA=y

Applying Pythagoras Theorem in Triangle AOB & Triangle OAC

$AB^2=OB^2+OA^2$ & $AC^2=OA^2+OC^2$

$(15)^2= x^2+y^2$ & $(20)^2= y^2+ (25-x)^2$

$x^2+y^2=225$ -------(i) & $400=(25-x)^2+y^2$ ---------(ii)

Now, (ii)-(i)

$[(25-x)^2+y^2 – (x^2+y^2)] = 400-225$

Or, $(25-x)^2 – x^2 =175$

Or, (25-x-x)(25-x+x)=175

Or, x=9

Put x=9 in (i)

$81 + y^2=225$

Or, $y^2=144$

Or, $y=12$

Therefore, OA=12cm, OB= 9cm

Now, Volume of double cone= volume of cone CAA'+ Volume of cone BAA'

$$=\frac{1}{3}\pi\,(OA)^2\times OC + \frac{1}{3}\pi\,(OA)^2\times OB$$

`

$$=\frac{1}{3}\pi\,(12)^2\times(16+9)$$

$$=3768\ cm^3$$

Now ,

Surface Area of the double cone= Curved Surface Area of the cone CAA'+. Curved Surface Area of the cone BAA'

$$= \pi\times OA\times AC \quad + \quad \pi\times OA\times AB$$

$$=420\,\pi$$

$$=1318.8\ cm^2$$

STATISTICS

LEVEL--1

Q1.

Classs Interval	No. of workers	X_i	f_ix_i
0 - 10	7	5	35
10-20	10	15	150
20-30	15	25	375
30-40	8	35	280
40-50	10	45	450
Total	50		1290

Mean = $\sum f_ix_i$ =25.8 $\sum f_i$

Q2.

Marks	cf
More than 50	35
More than 55	33
More than 60	27
More than 65	19
More than 70	5

Q3.

Marks	Cf
Less than 30	4
Less than 40	9
Less than 50	15
Less than 60	26
Less than 70	535

Q4.

3 Median = Mode + 2Mean

= 7.88 + 2(8.32)

3 Median =24.52

So, Median = 8.17

LEVEL – 2

Q1

Modal Class ---- 60 - 80

f1=20, f0=10, f2=12

L= 60 , h= 20

Mode = L + $\left\{ \frac{f1-fo}{2f1-fo-f2} \right\}$ x h

Put the values in the above formula and get mode = 71.11

Q2

Weekly wages (in Rs.)	**N0. of workers**	x_i	f_ix_i
40-43	31	41.5	1286.5
43-46	58	44.5	2581
46-49	60	47.5	2850
49-52	F	50.5	50.5f
52-55	27	53.5	1444.5
Total	176+f		8162+50.5f

Mean = $\frac{\sum f_ix_i}{\sum f_i}$

$47.2 = \frac{8162+50.5f}{176+f}$

8307.2 + 47.2f = 8162 + 50.5 f

145.2 = 3.3f

f = 44

Q3

Class	Frequency	cf
5-10	5	5
10-15	6	11
15-20	15	26
20-25	10	36
25-30	5	41
30-35	4	45
35-40	2	47
40-45	2	49

n=49 $\frac{n}{2}$ =24.5 Median class --- 15-20, L=15, f=15, cf =11, h= 5

Median = L + $\left\{ \frac{\frac{n}{2}-cf}{f} \right\}$xh

Put the values in the above formula and get the median = 19.5

Q4

Modal Class ---- 12 - 15

f1=23, fo=10, f2=21

L= 12 , h= 3

Mode = L + $\left\{\frac{f1-fo}{2f1-fo-f2}\right\}$ x h

Put the values in the above formula and get mode = 14.6

LEVEL -3

Q1

C.I	f	cf
0-10	5	5
10-20	X	5+x
20-30	20	25+x
30-40	15	40+x
40-50	Y	40+x+y
50-60	5	45+ x+y
Total	60	

Median = 28.5 Median class – 20-30 , L = 20, f=20, h=10, cf=5+x, n=60 , $\frac{n}{2}$ =24.5

45 + x + y = 60
x + y = 15------------(1)

Median = L + $\left\{\frac{\frac{n}{2}-cf}{f}\right\}$xh

Putting the values we get, x = 8
Put in (1) and get y =7

Q2.

C.I	f_i	x_i	f_ix_i
0-20	5	10	50
20-40	f_1	30	$30f_1$
40-60	10	50	500
60-80	f_2	70	70 f_2
80-100	7	90	630
100-120	8	110	880
	50		2060+30 f_1+70 f_2

30 + f_1+f_2=50
f_1+f_2= 20 -------------(1)

Mean = $\sum f_ix_i$ $\sum f_i$

Put the values in the above equation to get the equation
3 f_1 + 7 f_2 = 108 --------------(2)
Solving (1) and (2) ,we get
f_1 = 8 and f_2= 12

Q3.

Weekly	No. Of	Less than type	cf	More than type	cf

Wages	Workers				
0-20	40	Less than 20	40	More than 0	200
20-40	51	Less than 40	91	More than 20	160
40-60	64	Less than 60	155	More than 40	109
60-80	38	Less than 80	193	More than 60	45
80-100	7	Less than 100	200	More than 80	7

Draw ogive of both the types: -

Less than type - (20,40) (40,91), (60,155), (80, 193), (100,200)

More than type – (0,200), (20, 160), (40,109), (60,45), (80

PROBABILITY

LEVEL – I

1. Total cards = 52
   Cards of King = 4
   P (King) = $\frac{4}{52}$ or $\frac{1}{13}$
2. Sample space of tossing a coin = { Head, Tail }
   Sample space of throwing a die = {1, 2, 3, 4, 5, 6}
3. Probability of an event is p
   Then probability of complementary event will be 1-p
4. Sample space of throwing a die = {1, 2, 3, 4, 5, 6}
   Favourable outcomes (a prime number) = {2, 3, 5}
   P (a prime number) =$\frac{3}{6}$ or $\frac{1}{2}$
5. Sample space of throwing a die = {1, 2, 3, 4, 5, 6}
   Favourable outcomes (a number less than 9) = {1, 2, 3, 4, 5, 6}
   P (a number less than 9) =$\frac{6}{6}$ or 1
6. Sample space of English alphabet = {A, B, C……. Z}
   a. Vowels {A, E, I, O, U}
      P (Vowels) = $\frac{5}{26}$
   b. P(consonant) = $\frac{21}{26}$
7. Total cards = 52
   Cards of a red heart = 1
   P (a red heart) = $\frac{1}{4}$
8. Sample space of tossing a coin twice = { HH, HT,TH,TT }
   Favourable outcomes (at least one head) = 3 as {HH, HT, TH}
   P (at least one head) = $\frac{3}{4}$
9. Probability of a sure event is 1.
10. P (Event) = $\frac{Number\ \ of\ \ outcomes\ \ favourable\ \ to\ Event}{Total\ \ number\ \ of\ \ outcomes}$.

LEVEL – II

1. Sample space of throwing a die = {1, 2, 3, 4, 5, 6}
   Favourable outcomes (a number less than 4) = {1, 2, 3}
   P (a number less than 4) =$\frac{3}{6}$ or $\frac{1}{2}$
2. Sample space of tossing a coin twice is { HH, HT,TH,TT }

Favourable outcomes (Two tails) = 1

P (Two tails) =$\frac{1}{4}$ or $\frac{1}{2}$

P (At least one tail) =$\frac{3}{4}$

P (No tail) =$\frac{1}{4}$

3. Total cards = 52 Ace cards = 4 P (Ace cards) =$\frac{4}{52}$ or $\frac{1}{13}$

   Total cards = 52 Face cards = 12 P (Face cards) =$\frac{12}{52}$ or $\frac{3}{13}$
4. Total balls =5+8+4+7=24

   P (Black ball) =$\frac{7}{24}$, P (Red ball) =$\frac{5}{24}$, P (not green ball) =$\frac{20}{24}$ or $\frac{5}{6}$
5. Sample space of tossing a coin twice = { HH, HT,TH,TT }

   Favourable outcomes (exactly one head) = 2 {HT, TH}

   P (exactly one head) = $\frac{2}{4}$ or $\frac{1}{2}$

   Favourable outcomes (Almost one head) = 3

   P (Almost one head) = $\frac{3}{4}$

6. We know that total cases are 36

$$\left\{\begin{matrix} (1,1), (1,2), \ldots\ldots.(1,6) \\ (2,1), (2,2), \ldots\ldots.(2,6) \\ (3,1), (3,2), \ldots\ldots.(3,6) \\ (4,1), (4,,2), \ldots\ldots.(4,6) \\ (5,1), (5,2), \ldots\ldots.(5,6) \\ (6,1), (6,2), \ldots\ldots.(6,6) \end{matrix}\right\}$$

7. Total Cards =15 Even Numbers =2,4,6,8,10,12,14

   Favourable outcomes =7

   P (Even Numbers) = $\frac{7}{15}$

   Number divisible by 2 or 3 are 2,3,4,6,8,9,10,12,14,15

   P (number divisible by 2 or 3) = $\frac{10}{15}$ or $\frac{2}{3}$
8. Total balls =5+4+7=16

   P (White ball) =$\frac{7}{16}$

   P (neither Red nor white ball) =$\frac{4}{16}$ or $\frac{1}{4}$
9. Total tickets =35 Prizes = 10

   Probability of getting a prize = $\frac{10}{35}$ or $\frac{2}{7}$
10. Probability of her winning the first prize in a lottery = 0.08 Total tickets = 6000

    Number of tickets she bought = 0.08 × 6000 = 480

LEVEL – III

1. Total cases = 36
   a. A total of 7

      Favourable cases are (1,6), (2,5),(3,4),(4,3),(5,2),(6,1) i.e. 6

      Probability of getting a total of 7= $\frac{6}{36}$ or $\frac{1}{6}$
   b. A total of 11

      Favourable cases are (5,6), (6,5) i.e. 2

      Probability of getting a total of 11= $\frac{2}{36}$ or $\frac{1}{18}$

c. Doublets
Favourable cases are (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) i.e. 6
Probability of getting a Doublets = $\frac{6}{36}$ or $\frac{1}{6}$

d. 6 as a product
Favourable cases are (1,6),(2,3),(3,2),(6,1) i.e. 4
Probability of getting 6 as a product = $\frac{4}{36}$ or $\frac{1}{9}$

2. Total days in leap year = 366 Complete weeks = 52 Days left =2
Total days in week =7
Favourable cases (Sunday, Monday),(Monday, Tuesday),(Tuesday, Wednesday), (Wednesday, Thursday),(Thursday, Friday),(Friday, Saturday), Saturday, Sunday) i.e. 7
P(53 Mondays in a leap year) = $\frac{2}{7}$
P(53 Tuesdays in a nonleap year) = $\frac{1}{7}$
3. Total days in non leap year = 365 P(Same birthday) =$\frac{1}{365}$ P(Different birthday) =$\frac{364}{365}$
4. Sample space = { BBB,BGG,BBG,BGB,GBB,GBG,GGB,GGG }
   a. P(There is girl child in a family) =$\frac{7}{8}$
   b. P(There is at least 2 girl children) = $\frac{1}{2}$
   c. P(There is no girl child) = $\frac{1}{8}$
   d. P(There are 3 girl children) = $\frac{1}{8}$
5. Yes equally likely
6. Total items = 200 Rusted items 25+75=100 P(Rusted or a bolt)=$\frac{100+25}{200}$ or $\frac{5}{8}$
7. Total cards 52-8 = 44 P(Club) =$\frac{1}{4}$ P(an ace) = 0
8. Total cards = 52 P(king or a spade) =$\frac{4+13-1}{52}$ or $\frac{4}{13}$
P(a non spade) = $\frac{39}{52}$ or $\frac{3}{4}$ P(either a king or 10of heart) =$\frac{4+1}{52}$ or $\frac{5}{52}$
9. Total days in Feb 2000 = 29 P(Tom was born on 13th Feb) = $\frac{1}{29}$
10. P(letter selected is N in word EXAMINATION) = $\frac{2}{11}$

ANSWER KEY FOR SELF-EVALUATION QUESTIONS

REAL NUMBERS

Q1. $2 \times 3^2 \times 13$ Q2. 36 Q3. 999720

Q6 . $\frac{a+b}{2}, \sqrt{ab}$, Infinitely many rational and irrational numbers

POLYNOMIALS

Q1. $a = -\frac{3}{2}$ Q2. $x^2 - 6x + 4$ Q3. No Q4. $\frac{15}{4}$

Q5. $\frac{-1}{3}$ and 3 Q6. $\frac{-1}{2}, 3, -2, -1$ Q7. $a = 3$

A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Q1. $x = a, y = b$ Q2. $k = 3$ Q3. $x = 24, y = 11$

Q4. $(3,4), (-1,-4), (9,-4)$ Q5. $x = -1, y = 1$

Q6. 26, 24 Q7. Rs 6000, Rs 5250 Q8. 60km/hr, 40km/hr.

Quadratic equations

Q1. $-\sqrt{3}, \frac{-7}{\sqrt{3}}$ Q2. $\left[1, \frac{2}{3}\right]$ Q3. Real & Distinct Roots

Q4. $1, \frac{(a-b)^2}{(a+b)^2}$ Q5. 5 mins. , 8 mins. Q6. 20 days

Q8. 3 m, 4m, 5m Q9. $p = 7;\ k = \frac{7}{4}$

Arithmetic progression

Q1. -12,-17 Q2. 1625 Q3. $n^2 + 2n$ Q4.163

Q5. 42 Q6.18th, 837 Q7. 50

Q8. 82,350

COORDINATE GEOMETRY

Q1. a) $\sqrt{41}$ b) $(a+b)\sqrt{2}$ Q4. $\left(\frac{-1}{3}, 0\right), (\frac{-5}{3}, 2)$

Q5. Area= 0, they form a straight line, Yes points are collinear.

Q6.Yes, ||gm is a rectangle. Q7. $k = -16$ Q8. $4:3$

INTRODUCTION TO TRIGONOMETRY

Q2. A=10^0

SOME APPLICATIONS OF TRIGONOMETRY

Q3. 140.73m Q4. 28.92m

Circles

Q1. $\sqrt{AB^2 + QB^2}$ Q2. 6cm Q3. $3\sqrt{3}$cm Q5. 25^0

Q6. AD=7cm, BE=5cm, CF=3cm Q8. 60cm^2 Q9. r = 2cm

Area related to circles

Q1. 11cm,3cm Q2. 10cm, 4cm Q3. 157cm^2 ,78.5 cm^2

Q4. 16.8 cm Q5. 1254.96cm^2 Q8. 962.5m^2 ,1743.75m^2

Surface areas and volumes

Q1. 3:1:2 Q2. Rs. 10230/- Q3. 395.37kg

Q4. Rs156.89, Rs98.05 Q5. Rs. 440 Q6. Rs 65.312,Rs 24.492

Q7. 857.22cm^2 , 1950.67cm^3 Q9. 769.13cm^2 Q10. r=3cm, h=9cm

Statistics

Q1. 8.1 Q2. 167.05cms Q3. 65.625

Q 5. $x = 10$ Q7. X=34,$y = 46$ Q8. 7.675

Probability

Q1. $\frac{1}{3}$ Q2 a) $\frac{1}{26}$ b) $\frac{3}{13}$ c) $\frac{3}{26}$ d) $\frac{1}{52}$ e) $\frac{1}{4}$ f) $\frac{1}{52}$

Q4. $a)\frac{1}{6}$, $b)\frac{5}{6}$ Q5. a) $\frac{4}{9}$ b) $\frac{5}{9}$ c) $\frac{1}{3}$ d) $\frac{5}{18}$

Q6. . a) $\frac{1}{2}$ b) $\frac{1}{2}$ c) $\frac{9}{14}$ d) $\frac{9}{14}$ Q7.a) $\frac{2}{23}$ b) $\frac{5}{46}$

Q8. $\frac{3}{4}$ Q9. a) $\frac{8}{25}$ b) $\frac{2}{25}$ c) $\frac{11}{25}$

Q10. $\frac{3}{4}, \frac{5}{23}$